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Spreadsheets as cognitive tools: A study of the impact of spreadsheets on problem solving of math story problems

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Abstract In this study, we investigated the impact of computer spreadsheets on the problem solving practices of students for math story problems, and more specifically on the transition from arithmetic to algebraic reasoning, through the construction of algebraic expressions. We investigated the relationships among the students' prior knowledge and skills, the verification processes, and the effectiveness of the problem solving tasks. For identifying the factors involved in the problem solving process and their role, in our analysis we employed the Structural Equation Modeling (SEM) approach. We mainly focused on math story problems and on students of tertiary education with little prior experience on the use of computers and spreadsheets. Analysis of the data indicates that spreadsheets can support the transition from arithmetic to algebraic reasoning and this transition is influenced by prior skills of the students relevant to the interaction with the interface (enter formula skills), and the students' frequency of verification of the solution.

Keywords Problem solving · Spreadsheets · Cognitive tool · Story problems · Algebraic expressions · Solution verification · Structural Equation Modeling · Algebraic reasoning

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1 Introduction

This study is situated within the context of the investigation of the potential and support of software as tools for learning, and their links to the cognitive process of the students. We focus on the employment of spreadsheets in mathematics education for the construction of algebraic expressions and the solution of story problems. We investigate the impact of the affordances of the environment on the cognitive processes of students and the problem solving tasks. More specifically we focus on the process of solving a problem and verifying the solution and on the role of prior skills and knowledge relevant to the manipulation of the spreadsheet environment elements and the interaction with the interface.

Verification of solution and reasoning constitutes one of the most critical steps in the process of problem solving (Polya 1957; Jonassen 2000; Schoenfeld 1985, 1992; Mevarech and Kapa 1996). Before reaching their final solution, the problem solvers verify their findings for potential conflicts. If the outcome is perceived as erroneous, solvers re-track their steps, in the problem solving process, and check for mistakes in their reasoning or in the formulations of arithmetic expressions. Verification of results not only increases the rate of correct solutions, but also constitutes an important cognitive process for the comprehension and internalization of the transition from the arithmetic to the abstract reasoning. Previous research on verification, though, has mainly focused on error-detection of existing spreadsheets in work environments with little connection to the cognitive processes involved and the impact on learning and problem solving (Powell et al. 2008; Galletta et al. 1997; Teo and Lee-Partridge 2001). The versatility of spreadsheets, their cognitive benefits, and their widespread application for problem solving in both business and education settings were our main motivations for situating our investigation within the framework of computer spreadsheet environments (Johnston-Wilder et al. 2005; Jonassen et al. 2003; Jonassen 1996; Komis et al. 2006; Lavidas et al. 2007). Spreadsheets, as general purpose software, present additional advantages for education in relation to software specifically designed for solving problems (PBLEs) (Jonassen et al. 2003). In general purpose software, students do not have to cope with the additional cognitive load of having to learn the programming language of the software for the expression of reasoning. Furthermore, not any PBLE can be applied to any type of problem, in any thematic field, in different areas or with a variety of modes and formats, as is the case for general purpose software (Jonassen et al. 2003; Sutherland and Balacheff 1999; Abramovich 2003). This versatility renders general purpose software a more appealing, efficient and cost-effective solution particularly for education and school settings (Depover et al. 2007; Jonassen et al. 2003).

The potential of spreadsheets for education and learning more specifically has been recognised by a large body of research (Tort and Blondel 2007; Blondel et al. 2008). Spreadsheets constitute cognitive tools which may support the construction of knowledge within the appropriate learning context (Jonassen 1996). The multiple representations of a problem and the interactive manipulation supported seem to be some of the most effective for learning properties of spreadsheets. They can facilitate manipulation or speculation with numbers, they support decision making, and problem solving, and they are most effective in solving quantitative problems (Shim and Li 2006). The students can easily explore different representations of the problem -such as the numeral and algebraic representation, the table of values, the graphs and the functions- and they can identify their dynamic links and relations (Schwarz and Dreyfus 1993; Tort et al. 2008). The dynamic multiple representations of the problem and the interactive exploration of the content have the potential to scaffold understanding of the relevant mathematical concepts and the acquisition of knowledge (Hershkowitz and Kieran, 2001). Spreadsheets also have the potential to support the transition from concrete thinking to abstract thinking – the transition from the arithmetic reasoning to the algebraic reasoning as the "building of meaning for the symbols and operations of algebra in terms of arithmetic" (Kieran and Chalouh 1993), particularly in the case of math story problems (Friedlander 1998; Sutherland and Rojano, 1993; Sutherland 1993). The development of algebraic reasoning constitutes one of the main goals in mathematics education, as it provides the foundation for the comprehension of the abstract mathematical concepts (Kieran 2004).

Although research on spreadsheets and learning has focused on the identification of the functions that support algebraic reasoning and formulation during problem solving, research on functions and conditions that support verification of reasoning is still limited. Verification of reasoning is, as described, one of the most critical steps for effective problem solving in mathematics education. Our research focuses, therefore, on this aspect of problem solving in spreadsheet environments. Through the investigation of the factors and processes involved, possible benefits or drawbacks of spreadsheets will be identified and the full potential of spreadsheets as tools for learning will be exploited for the development of the relevant cognitive processes and the acquisition of knowledge by the students. Our focus is on the impact of spreadsheets on the effectiveness of problem solving activities for simple story problems and particularly on the process of the verification of the solution. For our study, we conducted a controlled experiment with students of Social Sciences at the Department of Educational Sciences and Early Childhood Education at the University of Patras.

2 Theoretical background

Problem solving constitutes a critical component, at the core of mathematics education (Branca 1980; Muir et al. 2008; Nunokawa 2005; Polya 1957). It has been identified as both a cognitive process and as an activity of higher learning significance (Jonassen 2004). Problem solving activities are more than a goal to be attained or a means of assessment; they are also activities through which the students can experience and learn mathematic concepts (Cai et al. 2005; Nunokawa 2005; Schoenfeld 1985). Story problems, more specifically, are the most commonly encountered problems within the school settings (Jonassen 2004). They usually contain not only textual information but also information in other representational forms such as images, figures, graphs and data tables. The information included is situated with a narrative context, often derived from every-day problems relevant to the experiences of the students. This context provides an additional motivation and a meaning for the solution of the problem (Jonassen 2004).

The problem solving process for story problems encompasses a sequence of steps the problem solvers follow involving both individual, cognitive factors and externalised actions (Polya 1957). The problem solvers will first have to analyse and comprehend the problem, form the relevant mental representations supporting the solution of the problem (Smith 2003), generate, analyse and select the best solutions, construct the relevant arithmetic or algebraic expressions, implement the solution by entering the formula in the spreadsheet cell, and finally assess and verify the solution. One of the most critical cognitive factors affecting problem solving seems to be the prior knowledge of the students, while external factors mainly address the interface and functionalities of spreadsheets as computer environments and their impact on the actions of the problem solvers and their interaction with the environment (see Fig. 1).

More specifically, prior knowledge plays a critical role for the comprehension of and the solution generation for a problem. It involves the existing knowledge and skills in the area relevant to the problem (content-specific knowledge) and the ability to analyze different information presented in math story problems (Relationship E in Fig. 1) (Funke and Frensch 1995; Jonassen 2000). These internal representations of the problem solver are externalized through the interactions of the solver with the computer environment (Jonassen et al. 2003).

Comprehension and internal representation of the problem are followed by the selection of the most appropriate solution. Particularly for math story problems, the solution consists of the development of a mathematical model - an algebraic expression using operators, symbols and numbers which connect the constructs of the problem (Kieran 2004; Galbraith and Stillman 2006; Johnston-Wilder et al. 2005). Solution generation and selection seems to be an important step in the problem solving process not only because the solution of the problem will be decided at this step, but also because this is the step the problem solver will have to return to in the case the final solution is not assessed as correct (Jonassen 2000, 2004) and repeat the process (Jonassen 2004; Mayer 1992). The solution generated and selected will then be implemented in the computer environment and verified by the problem solver.

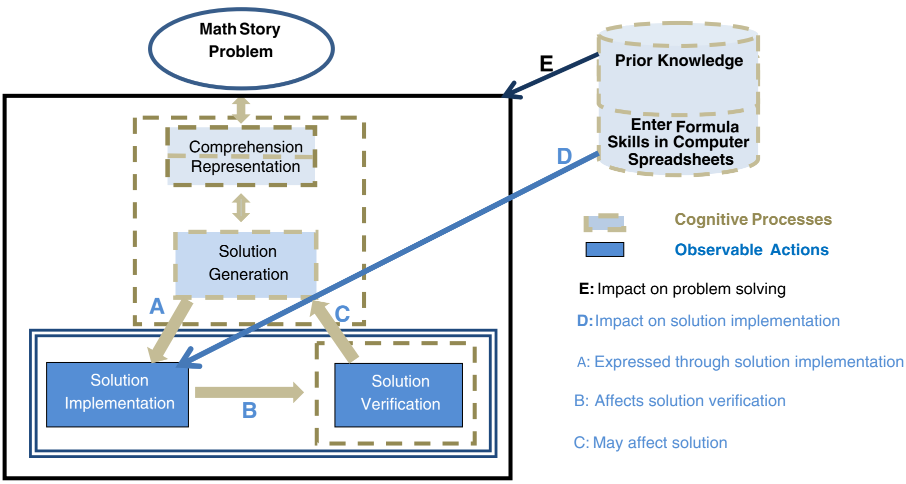

Fig. 1 Math story problem solving processes and interactions in a computer spreadsheet environment

In the solution implementation step, the problem solver has to enter the solution, in the form of an algebraic expression (Relationship A in Fig. 1). Previous knowledge and more specifically skills relevant to the insertion of the formula (enter formula skills) in spreadsheets are definitive for the accomplishment of this step. The solver will have to have acquired the relevant skills for being able to enter the appropriate formulas in the spreadsheet environment (Galbraith and Stillman 2006) (Relationship D in Fig. 1). Skills relevant to the insertion of formulas are also important for the utilization of the full potential of formula insertion functionalities of the environment and the verification of the solution, through exploration of the different aspects of the problem (Jonassen, et al. 2003).

In relation to the external factors involved in the solution implementation step, the "symbolic representation of data" and the "automation of computations" seem to be the main features of spreadsheets supporting students in their effort to implement their solutions (Abramovich 2003; Zacharos et al. 2007; Lavidas et al. 2007; Johnston-Wilder et al. 2005; Galbraith and Stillman 2006) and develop an algebraic expression (Kieran 2004; Sutherland and Balacheff 1999). The symbolic representation of data refers to the entering of formulas with the use of symbols, such as relative or absolute references to the cells involved, and also to the use of built-in functions. The automation of computations refers to the possibility of spreadsheets to automatically compute the results of the formulas during the insertion of data (Johnston-Wilder et al. 2005).

Assessment and verification of the solution are critical for the successful solution of a problem (Cai 1994; Eizenberg and Zaslavsky 2004; Polya 1957; Jonassen 2000; Schoenfeld 1985; Mevarech and Kapa 1996) (Relationship C in Fig. 1). Through the verification process problem solvers may identify reasoning and accidental errors (Powell et al. 2008) and further revise their proposed solution. The role of the computer environment features, in this process, is again important: upon insertion of the formula, the spreadsheet environment provides immediate feedback (Relationship B in Fig. 1). Feedback can be indirect, when the observed results do not agree with the expected results, or direct through environment error messages (Jonassen et al. 2003; Yerushalmy 1991). When solving a problem using pen and paper, the solver will have to manually conduct the calculations, as opposed to the automatic calculations of the computer environment. The automatic presentation of the results, instantly upon formula insertion, will present the final result or an error message. The impact of this immediate direct or indirect feedback seems to be stronger, in relation to the traditional pen and paper approach of solving problems (Papadopoulos and Dagdilelis 2008).

Even though the benefits of spreadsheets on problem solving and learning in mathematics have been recognized, most of existing, relevant studies refer to closed PBLEs and mainly to the support provided to the solvers (Jonassen et al. 2003; Papadopoulos and Dagdilelis 2008). Furthermore, there is little empirical evidence and data on the relation of the solution verification process with the effectiveness of problem solving, and on the impact of prior knowledge and skills relevant to formula entering on the verification process. Such evidence would provide us with valuable insights on the impact of specific computer environment features on the cognitive processes of the students, during problem solving, with possible further implications on the design of such environments and on the teaching practices in mathematics education.

3 Conceptual framework

In this study we mainly focus on the observable actions of the students relevant to the manipulation of the problem constructs and of the features of the environment in the two main steps of the problem solving process: the implementation of the solution and during the verification of the solution. Students were involved in tasks of simple math story problems. A problem is considered as simple when there is a correct solution consisting of a specific algorithm.

With the term *Effectiveness of Problem Solving in the Spreadsheet Environment (PS Effectiveness in Spreadsheet)* we refer to the degree of the construction of the mathematical model, the assessment of the final solution the student enters in the spreadsheet environment, by the researchers. Respectively, with the term *Effectiveness of Problem Solving using Pen and Paper (PS Effectiveness using Pen and Paper)* we refer to the researchers' assessment of the solution provided by the students, in the pen and paper condition.

The term *mathematical model* of problem solving is relevant to the algebraic mathematical formulas constructed by the students, so that these formulas represent and solve the specific problem (Tukiainen 1996).

With the term *Verification of Solution* we refer to the assessment and verification of the solution, that is the algebraic formula, and consequently the verification of the reasoning that lead to the construction of the specific formula.

In this study we also recorded the *Enter Formula Skills*, the ability of the students to enter formulas in the spreadsheet environment. More specifically, we recorded the ability of the students to enter algebraic formulas through the use of relative and absolute references and also the ability to apply the built-in functions SUM, AVERAGE, IF and the automatic completion of cells (use the fill handle to fill data), in the spreadsheet. The relative, mixed and absolute references are symbolic determinants of cells' positions and consist of the header of the row and the header of the column (Komis et al. 2006). These references in a formula define the relation of the cells described, with the cell which includes the formula. Any change in the values of these cells will also change the content of the latter cell which includes the formula.

4 Research questions

Our research objectives are: a) to identify the effect of the general purpose software computer spreadsheet environment on the effectiveness of problem solving for simple math story problems and b) to define the impact of entering formula and of the verification of the solution on the implementation of the solution of the mathematics story problem.

The research questions of our study are:

- Does the development of algebraic expressions differ in a spreadsheet environment and in a traditional, pen and paper approach?
- Do the skills relevant to formulas entering and the verification of solution process affect the development of algebraic expressions in a spreadsheet environment and at what extent?

5 Research methodology

The experimental phase was conducted during the course of four weeks, in the beginning of the spring semester at the department of Educational Sciences and Early Childhood Education at the University of Patras.

In the first week a questionnaire was given to the students for testing their prior knowledge and experience with computer spreadsheets (Computer Literacy Profile). The questionnaire was based on the Computer Experience Assessment Framework (CEAF) proposed by Palaigeorgiou et al. (2006). Through this questionnaire the knowledge and skills on computer use, acquired through teaching and through practice were assessed. We adapted the questionnaire to the objectives of our study by adding questions relevant to the experience of the students with problem solving, such as questions on their high school specialization (theoretical or technological), whether they enjoy solving mathematics problems, and whether they took exams on mathematics as a prerequisite for entrance in tertiary education.

The students initially took a 45-minute problem solving test with pen and paper. The students had to formulate the correct algebraic expressions for the solution of the problems. The problem solving test aimed at identifying the effectiveness of problem solving when using pen and paper. After the test, the students were asked to report whether and for which problems they had gone through a verification process (see Fig. 2 for the schema of the process and relevant data gathered).

During the second week, the students had to attend two two-hour sessions (teaching intervention) on the use of spreadsheets, and more specifically *MS-Excel 2003*. This intervention was part of the formal curriculum of their studies, for the course "*Introduction to ICT in Education*" and aimed at providing the students with the basic skills for formula entering in the spreadsheet environment, and more specifically the use of relative, absolute and mixed references, use of the fill handle for the data (automatic completion of cells), and use of the built-in functions (e.g. SUM, AVERAGE, IF). Through this teaching intervention we ensured that all the students have received the same training on enter formula skills and would further be able to interact with the spreadsheet environment.

On the third week, the students took the test of the assessment of knowledge and skills on basic formula entering functionalities, so as to examine the skills they had

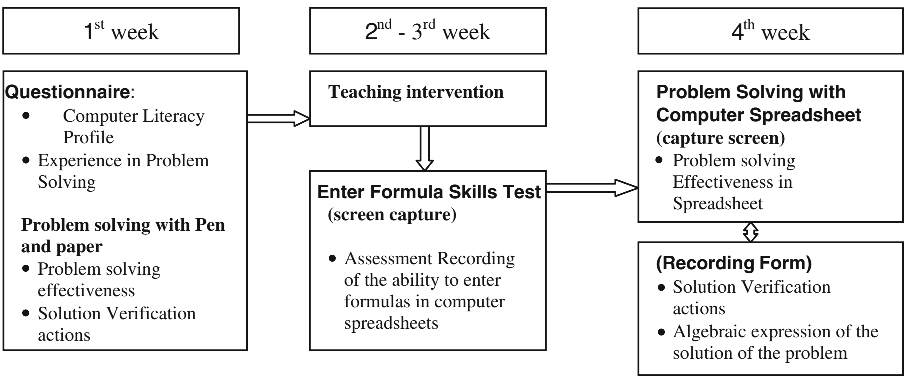

Fig. 2 Research process and relevant data gathered

actually acquired, following the teaching intervention. For recording the enter formula skills, the students had to complete a 45-minute test. The interactions of the students with the environment were recorded with the use of screen capturing software (*Camtasia 4*).

On the fourth week, the students took a 45-minute test of solving two simple story problems in the computer environment of spreadsheets. The actions of the students were again recorded (screen capture). During the problem solving process, the students had to complete a report form on their actions for the verification of their reasoning. Students were asked to report whether and for which problems they verified their solutions and also write the algebraic expression which was their proposed solution for each of the problems.

For both problem solving tests of the experimental phase (pen and paper and spreadsheet) the students had to work individually. The same problems were also employed in both conditions. These problems referred to calculus with percentages. More specifically:

Problem 1: Given the prices of the products, transportation cost, and taxation percentage, the students had to construct the algebraic expression for the calculation of the tax.

Problem 2: Given the number of votes for the three parties in student elections, the students had to construct the algebraic expression calculating the percentage of votes for each party.

Issues relevant to the comprehension of the problem statements were not considered, since one of the researchers and authors of this paper was present during the whole process providing clarifications in both experimental problem-solving tests.

The reports of the students on their verification process for both conditions (pen and paper and spreadsheet) were later compared and analysed, for the identification of possible differences between solution with pen and paper and in the spreadsheet environment. The verification reports were cross-examined with the screen capture videos. For identifying whether a student had actually performed a verification process the reports, the actions of the students and their interactions with the interface as recorded in the videos were all considered.

6 Participant characteristics

The student sample participating in the experiment consisted of 124 students in Social Sciences (Department of Educational Sciences), second and third year of studies (45% and 55% respectively), 19–20 years of age. The students participated in the whole experimental process. During secondary education they had all attended a theoretical direction of studies and were not examined in mathematics for their acceptance at the university. They reported that they enjoy "a little" or "not at all" to solve mathematics problems. They had little prior knowledge and experience in spreadsheets and had not used any spreadsheet software in the last three years. Students with a high level of prior knowledge of spreadsheets, as indicated by the initial questionnaire on the use of computers, were excluded from the experiment, so as to ensure homogeneity of the sample.

7 Research variables

In the SEM analysis the following variables were identified:

7.1 Observable variables

a) The assessment of the effectiveness of the solution of both problems, both for the pen and paper condition and the spreadsheets environment (*PS Effectiveness using Pen and Paper – problem 1, PS Effectiveness using Pen and Paper-problem 2* and *PS Effectiveness in Spreadsheet – problem 1, PS Effectiveness in Spreadsheet – problem 2*),
b) *Enter Formula Skills*: assessment of the skills of the students on entering formulas in the spreadsheet environment. The *assessment of the skills* of the students on formula entering in the spreadsheets was based on the test that took place after the teaching intervention, and referred to the ability of the students to enter formulas by using a) the relative references, b) the absolute references, c) filling in the cells, and d) the SUM, and AVERAGE functions. For the assessment, the screen capture videos and a four-point scale evaluation form were used: 1 (the student did not performed any action in relation to the problem), 2 (the student did not accomplish the goals required), 3 (the student types numbers in the cell and not a formula), 4 (the answer of the student is correct but the process includes a sequence of errors and revisions), 5 (the answer of the student is correct, with no errors and revisions). The average of the assessment of the actions in the test produced the variable *Enter Formula Skills,* and,
c) *Verification of Solution*: the frequency of solution verification processes during the problem solving tasks in the spreadsheet environment. Concerning the *Verification of Solution* during the problem solving process in the spreadsheets, the students had to keep notes of their actions relevant to the verification of their solution and reasoning in the recording form provided. For identifying the values for this variable, we analysed the qualitative data (video recordings, reporting forms). Findings from the videos were cross-examined with the reporting forms. For the students that had gone through the process of verifying their solutions, a dual variable was used (1=the student verified the results, 0=the student did not verify the results) and the total frequency of solution verifications for all the problems, for each student, was calculated. In our analysis, the variable *Verification of Solution* was considered as an observable variable.

7.2 Latent variables

The latent variables in our analysis are 1) *PS Effectiveness using Pen and Paper* and 2) *PS Effectiveness in Spreadsheet.*

For identifying the latent variable *PS Effectiveness using Pen and Paper* two observable variables were employed: a) *PS Effectiveness using Pen and Paper – problem 1*, and b) *PS Effectiveness using Pen and Paper- problem 2*. The solution of each problem mainly consisted of the construction of an algebraic expression (formula) linking the given facts of the problem. The effectiveness of solution was based on a four-point scale evaluation form of the solutions presented by the students: 1 (the student did not work on the problem), 2 (the solution proposed by the student is not correct), 3 (only the outcome or the distinct calculations of the problem solving process are presented by the student), 4 (the algebraic expression used is described by the student).

Similarly, for the identification of the latent variable *PS Effectiveness in Spreadsheet* the following two observable variables were used: a) *PS Effectiveness in Spreadsheet – problem 1*, and b) *PS Effectiveness in Spreadsheet – problem 2*. The assessment was based on the screen capture videos of the student-spreadsheet interactions in the computer environment. A five-point scale was again used: 0 (the student did not work on the problem), 1 (the solution presented by the student is not correct), 2 (only one arithmetic solution seems to be proposed by the student), 3 (the final algebraic solution of the student is correct – a sequence of student mistakes and revisions is also observed in the video), 4 (the student proposes the correct algebraic solution with no mistakes and revisions).

All the variables described above are considered ordered-categorical variables and may be included in a path analysis model within the framework of the Structural Equation Modeling approach (Kline 2010; Raykov and Marcoulides 2006). For our analysis, the Bayesian estimation was employed (Bolstad 2004) and the software *AMOS* was used (Arbuckle 2007).

8 Data analysis

Our initial model proposed included four variables: the latent variables *PS Effectiveness using Pen and Paper* and *PS Effectiveness in Spreadsheet* and the observable variables *Enter Formula Skills* and *Verification of Solution* (Fig. 3). Based on the analysis, it was concluded that the model tested fits the data (posteriori predictive p=0.15).

From the initial analysis, it emerged that the direct impact of a) the variable *Enter Formula Skills* on the variable *PS Effectiveness in Spreadsheet* and b) of the *PS Effectiveness using Pen and Paper* on the *Verification of Solution* are of no significance, and therefore these links were removed from the model and the analysis was repeated.

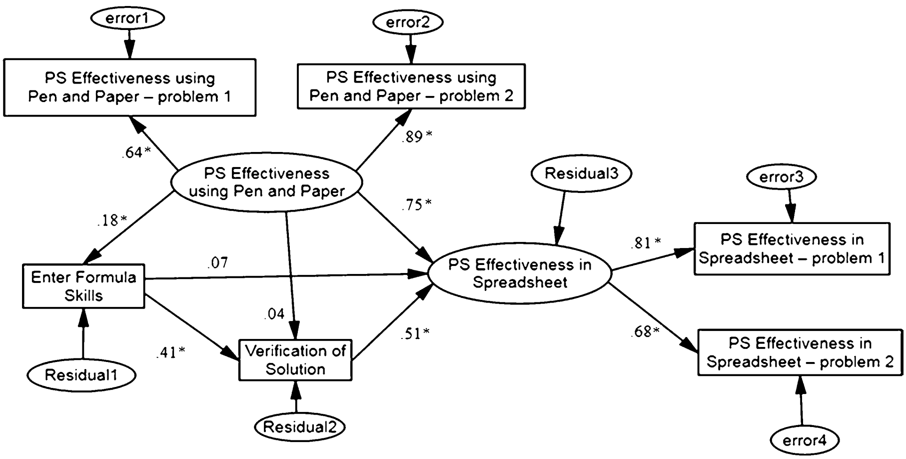

Fig. 3 Path model of the *PS Effectiveness in Spreadsheet.* (*statistically significant direct influences for significance level .05)

9 Presentation of the pruned equation model

The final pruned model (Fig. 4) seems to fit the data (posteriori predictive p=0.15). This model was therefore approved and can be employed for the interpretation of the effectiveness of *PS Effectiveness in Spreadsheet*, in relation to the factors that we focus on.

The statistically significant regression and correlation coefficients for all the factors involved in the model are presented in Table 2 (see Appendix). The direct and indirect influences, as presented in Fig. 4, will be further discussed. In this model, the direct and indirect influences of the variables on the *PS Effectiveness in Spreadsheet* are also presented (see Appendix Table 1).

More specifically, we observed the strong and direct impact (0.751) of the *PS Effectiveness using Pen and Paper* on the *PS Effectiveness in Spreadsheet*. With the level of the *Enter Formula Skills* and *Verification of Solutions* stable, the *PS Effectiveness using Pen and Paper* seems to have a strong impact on the *PS Effectiveness in Spreadsheet*. There is also an indirect impact on the *PS Effectiveness in Spreadsheet* (0.036) mediated by the *Enter Formula Skills* and the *Verification of Solutions*. The results of the students in the pen and paper condition and in the computer spreadsheet condition indicated that the main differences involve the construction of the algebraic expression. More specifically, most of the students in our research (problem 1: 13 of 18 students and problem 2: 16 of 20 students) who provided the solutions, in the pen and paper condition, using simple, distinct calculations, constructed the correct algebraic expression in the computer spreadsheet condition. This finding is supported by the fact that most of these students (problem 1: 13 of 13 students and problem 2: 15 of 16 students) wrote the correct algebraic expressions in the accompanying reporting form. It seems, therefore, that the computer spreadsheet environment can facilitate the transition from the arithmetic to the algebraic formulation of solutions.

There is no direct impact of the *ability to enter formulas (Enter Formula Skills)* on the *PS Effectiveness in Spreadsheet*. Any impact of the *Enter Formula Skills* on the *PS Effectiveness in Spreadsheet* may be explained by the *Verification of Solution*.

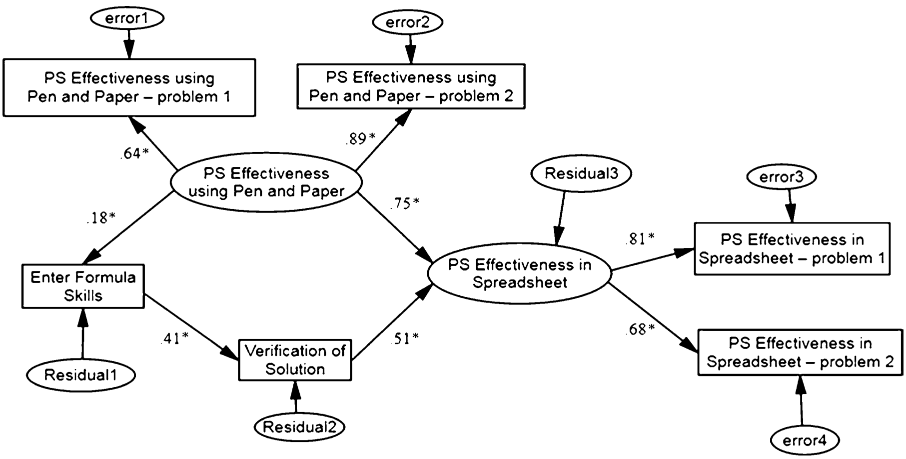

Fig. 4 Pruned structural model of the *PS Effectiveness in Spreadsheet.* (* statistically significant direct influences for significance level .05)

The impact is, therefore, indirect (0.209). From the analysis of this path, the following observations can be made:

a) Direct impact (0.409) of the ability of the students to enter formulas in the spreadsheet on the verification of solutions. It seems that when the students can enter formulas in the computer spreadsheets, they test their solutions more often (verification). There was a difference between the number of students who verified their solutions in the pen and paper condition and in the computer spreadsheet condition, for both problems. In the pen and paper condition 15 students verified their results for Problem 1, and 11 students for Problem 2. In the spreadsheet environment 56 students verified their results for Problem 1, and 58 students for Problem 2. Most of the verification processes, as it was observed in the videos, involved interaction of the students with the environment and employment of its features. The students, for example, were activating the cells relevant to the algebraic expression and entering different numbers in them, usually of a smaller value, in order to verify that the formula is correct. In other cases, they would enter different algebraic expressions in order to perform numeric calculations. More specifically, 59% of the verifications of the students for Problem 1, and 62% of the verifications for Problem 2 involved interaction with and employment of the features of the environment. It seems, therefore, that the computer spreadsheet environment facilitated verification of the solutions by the students more than the traditional pen and paper environment.
b) Direct impact (0.51) of the *Verification of Solution* on the *PS Effectiveness in Spreadsheet*. This finding seems to indicate that the more the students verify their results and solutions, the more correct their solutions are. This finding was supported by the cross-examination of the students' report forms and their solutions. The students who reported that they had verified their results had in most of the cases given the correct answer. 75% of the students, for Problem 1, and 86.2% of the students for Problem 2, who had verified their results, had also constructed the correct algebraic expression.
c) Finally, there seems to be a small direct impact (0.183) of the *PS Effectiveness using Pen and Paper* on the Enter Formula Skills. The students, therefore, who solved the problems on pen and paper, having constructed the correct algorithm, could also enter the correct formulas in the spreadsheet and use the signs of the arithmetic operations (+, -, *,/). The small impact (0.183) can probably be explained by the specialized knowledge required for entering formulas in the spreadsheets.

10 Discussion and conclusions

In this study we investigated the impact of the computer spreadsheet environment on the problem solving processes in mathematics education, and more specifically on the formulation of algebraic expressions as an aspect of algebraic reasoning, in relation to existing skills and previous knowledge of the students. Our results indicated that students using computer spreadsheets verify their solution more often than in the pen and paper condition. These results seem to be in a similar vein with the findings of Papadopoulos and Dagdilelis (2008). They compared computer environments for dynamic manipulation of geometric shapes for the solution of geometric problems to the conventional pen and paper process.They tested primary school pupils and found that the functionalities of the computer environment support solution verification.

Our findings also indicated that the computer spreadsheet environment facilitated the transition from arithmetic to algebraic reasoning. Students who had proposed solutions consisting of distinct, simple calculations, in the pen and paper conditions, solved the same problems in the computer spreadsheet environment using algebraic expressions. The formulation of algebraic expressions constitutes a critical step for the transition to algebraic reasoning (Sutherland & Balacheff, 1999). Similar research also concluded that computer spreadsheets facilitate transition from arithmetic to algebraic reasoning (Friedlander 1998; Sutherland & Rojano 1993).

The two main factors we focused on – formula entering skills and verification of the solution – seem to explain the effectiveness of the solution in computer spreadsheets. More specifically:

a) *enter formula skills in spreadsheets* seem to have an impact on the *frequency of solution verification*. Students, who had acquired the relevant skills for entering formulas in the spreadsheets, verified their solutions more often than those who had less developed formula entering skills. The symbolic representation of the data and the automation of computations in the computer spreadsheets seemed to support formula entering. We can, therefore, assume that this functionality supports the verification of their reasoning, as it allows the students to see the progression of the computations on the screen, relieves them from having to go through complex computations and provides them with the possibility to test their solutions (Jonassen et al. 2003).
b) *The frequency of verification* of solutions seems to impact on the *effectiveness of the problem solution* in computer spreadsheets. Our findings agree with the findings of Cai (1994) and Eizenberg and Zaslavsky (2004) who studied the problem solving process of complex mathematical problems. Eizenberg and Zaslavsky (2004) found that verification of reasoning only had positive results when the solver was informed whether the solution was correct or not. Kapa (2002) also had similar findings. She found that the metacognitive support of the computer environment during the problem solving process is related to the successful solution of the problem. It also seems to affect the outcome of the problem solving process, especially for solvers with little prior problem solving experience. Our results indicate the emergence of an interconnection of the formula entering skills, the verification process, and the effectiveness of the solution, which seems to facilitate the transition from arithmetic to algebraic reasoning.

For being able to construct algebraic formulas and verify their solutions, it seems essential that the students acquire at least the basic skills for the manipulation of spreadsheet functionalities, and formula entering skills more specifically. Such skills play an integral part for the exploitation of the full potential of spreadsheets, the effectiveness of the problem solving, and consequently the cognitive processes and activities of the students. Formula entering skills should not, therefore, be viewed as merely a functional competence for interacting with the computer environment. In mathematics teaching, when computer spreadsheets are employed, acquisition of skills relevant to the entering of formulas in the spreadsheet should also be emphasized (Beaman et al. 2005; Bostrom et al. 1988).

Our study mainly focused on undergraduate students. Previous research in the area involved primary schools students, who have mainly acquired arithmetic reasoning, or secondary school students, in the process of acquiring algebraic thinking (Lins & Kaput, 2004). Our sample, although already in tertiary education, did not seem to have fully acquired the skill to formulate their reasoning through algebraic expressions. For having a homogenous sample, we further selected students, who had followed a theoretical direction of studies, and therefore had less exposure to mathematics problems over the last two years of secondary education.

Our study is, certainly, limited by the small sample size (124 students) and their specific profile (students of social sciences, little experience with mathematics problems and spreadsheet use). We also focused on story problems and examined the performance of the students on two simple math story problems. Further studies with larger sample sizes, students of different backgrounds, and complex mathematics problems could benefit research on the impact of computer environments on learning and cognition, and also mathematics education and the teaching of mathematics through spreadsheets and similar software.

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Appendix

Table of the analysis results

Table 1 Standardized direct, indirect and total effects of pruned model

	R^2	Effect	PS Effectiveness using Pen and Paper	Enter Formula Skills	Verification of Solution
Dependent construct					
PS Effectiveness in Spreadsheet	.76	Direct	.751	0	.51
		Indirect	.036	.209	0
		Total	.787	.209	.51
Verification of Solution	.17	Direct	0	.409	
		Indirect	.074	0	
		Total	.074	.409	
Enter Formula Skills	.04	Direct	.183		
		Indirect	0		
		Total	.183		

Table 2 Regression weights and standardized weights (pruned model).

Latent construct Pair:			Regression, (Standardized) Weight	SD	CR*
PS Effectiveness using Pen and Paper	→	PS Effectiveness in Spreadsheet	.746(.751)	.189	3.947
PS Effectiveness using Pen and Paper	→	Enter formula Skills	.520(.183)	.235	2.21
Enter formula Skills	→	Verification of Solution	.611(.409)	.227	2.69
Verification of Solution	→	PS Effectiveness in Spreadsheet	.135(.510)	.043	3.139
Latent Indicator Pair:					
PS Effectiveness using Pen and Paper	→	PS Effectiveness using Pen and Paper – problem 1	1.374(.640)	.475	2.892
PS Effectiveness using Pen and Paper	→	PS Effectiveness using Pen and Paper – problem 2	1(.893)		
PS Effectiveness in Spreadsheet	→	PS Effectiveness in Spreadsheet – problem 1	1.227 (.814)	.319	3.846
PS Effectiveness in Spreadsheet	→	PS Effectiveness in Spreadsheet – problem 2	1(.678)		

*CR=Weight/SD

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