Mathematical concepts in Preschool Education. An attempt to teach the concept of "measuring" the volume of containers[1]

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1. The theoretical framework

1.1 The concept of measurement: psychological approaches and didactic perspectives.

The ability of measuring, in piagetian researches is ascertained by the comparison of two quantities. According to J. Piaget and his collaborators "measure is to take out of a whole an element, taken as a unit, and to transpose this unit on the remainder of a whole: measurement is therefore a synthesis of *sub-division* (of the whole) and *change of position* (of the selected unit)" (Piaget et al. 1960, p. 3). The comments in the brackets and the italics belong to the writer of this article). This procedure, although it may seem simple at the stage of the final equilibration, is in fact a result of a complex genetic process.

The acquisition of the measuring concept, in the piagetian theoretical context, will be built at the stage of a child's intellectual development, which is characterized by the use of a mediated measuring tool. It concerns children of 7+ ages, and a distinctive characteristic of this stage is an operational use of the measuring procedure which is expressed by general logical operations, like for example: if A=B and B=C, then A=C, or if A>B and B>C, then A>C.

However, there is a large number of experimental facts, both from the field of the psychology of learning, as well as the approaches from the field of the didactics of mathematics, which shows that children who begin school, don't have such limited abilities concerning mathematical concepts, as Piaget claims. These researches stress that the presentation of the mathematical abject in a way that males sense to the child,

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the provocation of social interaction and social-cognitive clash within the limits of the school class, as well as the social significance which conveys the educational material, are factors, the contribution which is significant to the learning process [Bishop, 1983; Doise & Mugny, 1981; Donaldson, 1991; Hughes, 1986; Pepper & Hunting, 1998; Perret-Clermont, 1986; Sierpinska & Lerman, 1996; Vygotsky, 1962, 1988; Wagman, 1975; Zacharos, 1999; Zacharos & Ravanis, 2000].

1.2 From natural objects to the mathematical concept of measurement.

In the first school grades, such as preschool and first school grade, it's important for children to be introduced to a big variety of environmental objects and materials, for which measurement can come up as a matter for speculation. Measurement offers us possibility to approach maths in a natural way.

The familiarization with the process of measuring, takes a different form according to the educational needs and the students' cognitive level. A child in Kindergarten can compare weights, lengths, quantities etc., without the need to measure or enumerate. During all these activities the child must be prompted to proceed to estimations. Our objective at this educational stage is not consider that the accuracy of an answer should necessarily include more enumeration. Moreover, such a didactic approach offers the potentiality of gradual familiarization with the idea of the "mediator" [HMI Series: Matters for Discussion 9, σ. 43], meaning the tool which is involved in the measuring process.

There is a similarity in all the cases of measurement. However, the principles of measurement could be more obvious in some cases than the others: for example, we can add two lengths directly, placing the one next to the other, edge to edge, something that can't be done for temperatures. We can also classify four children, and refer to their heights, but the comparison of three objects in terms of their capacity requires logical conclusions, which presuppose the acquisition of the transitivity property.

1.3 Volume and quantity: two distinctive concepts.

Despite the fact that Geometry considers that the concepts of length, area and volume are closely related, from the standpoint of psychology the acquisition of the volume concept, brings up particular problems, as it is connected more with the natural structure of objects and their three dimensional characteristics.

Furthermore, from the psychology standpoint there is a distinction between two invariant relating to volume: the one refers to the conservation of quantity and the other to the conservation of volume. On the contrary, Geometry doesn't make the distinction between these two invariant [Piaget 1960, p. 354]. The acquisition of the quantity concept and the concept of volume, require different periods of cognitive development. In this way, according to Piaget (1960) the conservation of quantity appears quite earlier than the conservation of volume. Based on the remarks mentioned above we judge that it would be didactically fertile to approach the volume of an object as the total space "occupied" or enclosed by the object [Piaget 1960, Bell, et. al. 1975]. In this case, what is used is the mediation of natural objects, as for example, three-dimensional objects (cubes, etc.) or materials which have a fluidity and can fill the interior of the objects being measured (water, sand, etc.).

In our research referring to preschool children, we will explore the possibility of introducing the volume concept approached through the capacity of containers. Moreover, we will proceed to measuring the capacity of three-dimensional objects. Specifically our didactic approach includes:

- The direct comparison of the capacity of containers, where we limit our research to quality characterizations such as "more" and "less".
- The indirect comparison of the capacity of containers. In this case, we attempt to measure their contents using "units" we will introduce. The measurement will take place through the process of enumerating the "units", a fact that gives a specific form to the numbers arising from the measurement.

2. The method

*2.1 Subjects and proces*s

Eighteen infants (around the age of five) from a public Kindergarten in Athens (Greece), participate in our research.The teacher of the class, who cooperates with the writer of this article, carried out the teaching procedure. The writer is in the classroom, monitors and records the children's actions. This activity takes place in a Kindergarten classroom in the presence of all the children. We place the materials on

a small table and invite the children individually or in pairs, according to the type of the activity. The children who aren't participating are sitting and watching.

For the analysis of our facts, we used a quality method of evaluating the children's actions, a fact that was facilitated by the videotaping of the activity. Our quality annotation includes both, spotting the ability of the children to invent ways to compare the capacity of the containers, as well as to stress the didactic obstacles, resulting either from the children's cognitive level, or the difficulty of dealing with the matter. Finally we record the teacher's contribution to teaching procedure and the way she handles the activity.

2.2 Teaching materials

- A wide and open container (disc).
- Different containers in shape and content.
- Cards showing the containers of figure 2.
- A die which on its three sides has the word "more" written and the word "less" on the other sides.
- A string-puppet representing a Chinese man.

3. Progression of the activity and annotation of the results

The scenario (the realization of the scenario took two days)

The Chinese string-puppet "Chan Chin" has visited the Kindergarten and brings the news from faraway China. It introduces the children to an atmosphere of comparison. The rice and the disagreements over its fair sharing will form the basis on which the scenario will be structured through all the stages of the activity. The activity progresses in such a way, that the learning demands are differentiated.

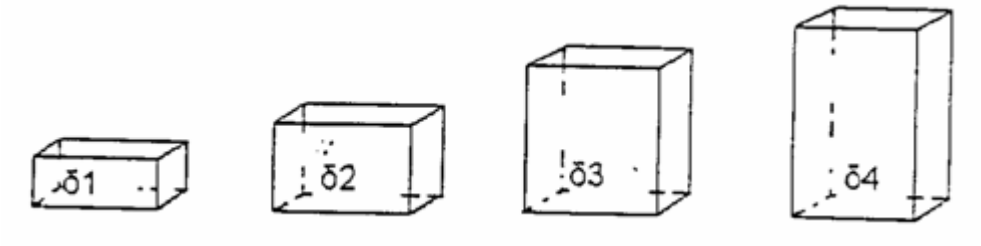
3.1 Stage A'. Direct comparison (This activity takes place the first day)

3.1.1 Comparison of containers which are different in one dimension

Our objective in this case is children's familiarization with the material and the comparison process. Our intention is to create a didactic atmosphere and a context framework, so that the children's actions are placed within the activity limits and its learning aim.

The containers that are used here, are the rectangular parallelepipeds in figure 1 which have equal bases and differ in height. Children are asked, one by one, to choose a container and fill it with rice. Then they select one of the empty containers. We ask: "*which of the two containers takes more rice?*"

Because the differences among the containers are obvious, the children select correctly the largest container. Afterwards they are asked to verify their selection, pouring the contents of the full container into the empty one. The questions usually asked by the teacher are: "*Will this container* (the empty one) *take all the rice?*", "*Will any be left* (inside the full container)*?*"

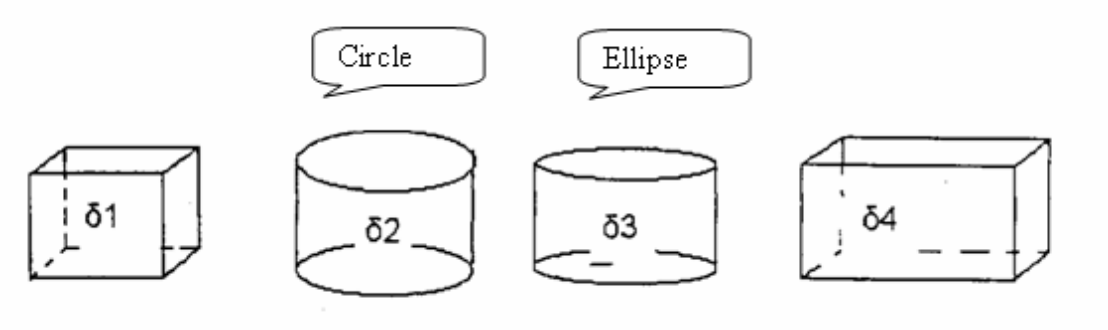


C1<C2<C3<C4

Figure 1

3.1.2 Containers of the same height and capacity but of different shapes

Here the containers have the same height and capacity but their shapes vary (fig. 2).



C1<C2=C3<C4

Figure 2

The children are asked to compare two containers, suggested by the teacher, one of which is full. Generally they respond to this activity too. Having observed the children's answers we point out the following:

1. What is noticed, is a concentration of their attention on the level of the rice inside the container. For example, during the comparison of the containers C2 and C4 (C2 is full) and after emptying the content of C2 into C4 Dimitris claims that:

> Dimitris: *This one* (C4) *takes more.* ***The rice is less****; there is a gap. Put some more to fill it, there is some missing.*

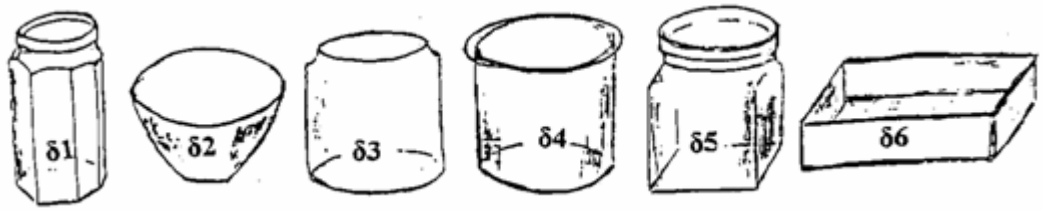
Of course the expression "*the rice is less*" could probably be due to the concentration of their attention one the level (in height) of the mediating material, a fact which refers to the concepts of conservation of Piaget.

2. In our theoretical references we underlined the need for a distinction between the concepts of volume and quantity. This fact is brought up in our research too: For example after an infant has ascertained that C4 can take three times the content of C1, he is asked how many times the small container can fit in the big one.

The child puts the small container (C1) inside C4 and pronounces: *Probably two times!*

3.1.3 Comparison of containers different in shape and dimensions (The game with the cards and the cube)[2]

Our objective at this stage was the need of comparison to arise from the need to respond to the rules of a game. Here we use six containers of different shapes and sizes, as shown in figure 3 (the containers shown in figure 3 are placed according to their capacity from the smallest to the largest).



C1<C2<C3<C4<C5<C6

Figure 3

[2] We got the idea for this activity from Cook, et. al. (1997).

We also have cards, which show each container and are placed upside down on the small table. Each child draws a card and seeks the containers it shows. Afterwards one of the children throws the die with the indications "more" and "less" on its sides. If the die shows "less", then the player having the container of the least capacity wins. If it shows "more" the player having the container of the largest capacity wins. The children compare the containers, according to their capacity in rice filling each one up and then emptying it into the other. In the end, the cards are out back in their places and are shuffled. We then record our remarks, arising from the analysis of the children's answers:

1. The form of the game seems to generally motivate the children in order to respond to the need of the comparison. Furthermore, the consent of the two players, as well as the other attending children, contributes positively to the choice of the right answer and its justification.
2. The mediation material (the rice) doesn't seem to help some children at first, during the comparison, since they concentrate on the level of the rice inside the container. In this case, the concept of "more" seems to be identified with the fact of whether the container is full or not. However after the auxiliary interventions of the Kindergarten teacher, they generally respond to other forms of this activity.

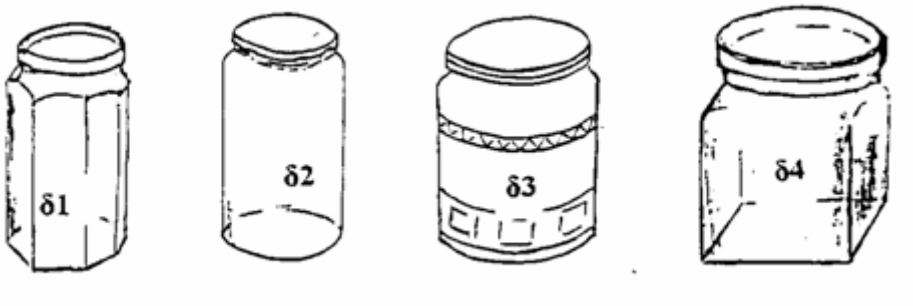
3.2 stage B': Indirect comparison: Measuring attempt.

3.2.1 preconditions for the successful comparison

The indirect comparison requires an extra didactic effort to the pint that it involves the introduction of a common mediator (a fact we also pointed out in the theoretical remarks), as for example the use of the ruler in order to compare two lengths, of a container which is used as a measure of comparison in capacity, etc. The conclusions here usually have the form: If A=B and B=C, then A=C, or if A>B and B>C, then A>C. In this cases the comparisons are indirect and take place with the combination of two different comparisons (for example A=B, B=C), where the compared quantities A and C are compared with quantity B (which acts as an intermediary). Furthermore, the successful comparison in this case presupposes the acquisition of the comprehension and the use of the measuring unit. This fact offers great potentialities to the process of measurement. The transitive entailing form a general precondition, necessary for a successful measurement. By the use of the

measuring unit though, we don't simply limit our speculation to transitive entailing, but, we can pronounce, for example, how many times quantity A is bigger than quantity C [Nunes, et. al. 1996].

Our objective here is to examine the possibility of comparing two containers, as far as capacity is concerned, without having the possibility of direct comparison. The compared containers (fig. 4) are filled with rice and the comparison takes place with the mediation of small similar containers, which according to the scenario are bowls from which little Chinese children eat their rice. The Chinese man, who brings the news from distant China, describes the disagreement between his two neighbors, about the quantity of rice each one of them got. The one with the largest family believed that his container had less rice than his neighbor's who had a smaller family.



C1<C2<C3<C4

Schema 4

3.2.2 qualitative annotation of the results

1. The pupils generally seem interested in solving the problem about the rice quantity difference. Some of them alone resort to "measurement" and empty the content of each container into the "food bowls". Then they enumerate and compare with success.
2. A number of children is content with optical comparison. These children are urged by the teacher to count how many Chinese children will eat the rice from each container. For instance Apostolis is asked to compare containers C1 and C2. At first he believes that container C1 "*is larger in capacity*". Further to that he is urged on by the teacher:

T (Teacher): *How many children will eat from this one* (C1)*?*

A (Apostolis): *Two* (after emptying its content into the bowls).

T: *How many children will eat from the other one* (C2)*?*

A: *Four children will eat from that one*.

T: *Which container will the Chinese man with the biggest family take?*

A: *This one* (C2).

Similarly another infant, Athina, is asked to compare the contents of containers C1 and C3.

T: *Which of the two containers do you think the Chinese man with the biggest family should take?*

A (Athina): (Hesitant) *This one, I think* (C3).

T: *Can we find out which one haw more rice?*

She empties the contents of her containers into the bowls and discovers it takes four bowls for C2 and five for C3.

T: *Which container should the Chinese man with the biggest family take?*

Athina: *The one with the five bowls*.

T: *Does this one* (C3) *take more rice?*

Athina: *Yes.*

T: *Which of the two numbers is bigger, four or five?*

Athina: *The four.*

T: *And bigger?*

Athina: *Number five.*

Closing the annotation of the results from our research we would like to stress the following:

1. The social significance which the didactic material involves, doesn't always contribute positively to our didactic aim, especially in cases where the social significance is relative to the learning subject. For example, an infant was asked to compare the contents of two containers, according to their capacity in beans. We have the containers C2 and C4 of figure 2. We fill up C2 with beans, empty them in C4 and fill up again C2. The infant concludes correctly that C4 takes more beans. However, his opinion about the content of the containers changes when the scenario is altered.

Teacher: *Who is yours best friend?*

Child: *Helen.*

Teacher; *If you eat beans from this container* (C4) *and your friend Helen from the other one* (C2), *who will eat more?*

Child: *Helen will.*

Teacher: *Why?*

Child: *Because I don't like beans in the first place!*

Also during stage B' of the activity, an infant, when urged to help the Chinese men find the biggest container in capacity, by-passes the matter of disagreement and suggests to us, to give an extra container with rice to the Chinese man with the biggest family.

2. The focus of the children's interest, on some dimension of the containers (usually in height) seems to be strong. In this way they sometimes hesitate, when they have to select between the height and the capacity of the containers. For example, a subject, while emptying the content of container C3 into C6 (fig. 3), when the differences in capacity are obvious, claims that C3 is larger in capacity because "*it's taller*". Only after our question "*is this container* (C6) *full?*" does the subject seem to realize the difference of the two containers in capacity and finally pronounces that C3 is smaller in capacity.

4. Discussion

Our intention here was to introduce the children to a difficult to comprehend geometrical characteristic of the three-dimensional objects, volume. We considered didactically advisable to approach this concept, through the concept of capacity.

The children dealt with the activity throughout all its forms and generally responded satisfactorily, sometimes alone and sometimes according to the teacher's indications. Nevertheless the duration of the activity seemed to tire them. What may have strengthened this, is the fact that, although the activity takes place in the Kindergarten classroom, children participate individually or in pairs, while the rest watch. It's a fact, that kindergarten in terms of buildings and organizing, don't offer the appropriate preconditions for the massive participation of the children in similar activities.

Furthermore, the time this teaching procedure was held, at the beginning of the school year (end of October), didn't give the children the possibility to appropriate concepts, which would facilitate a more effective dealing with the activity, such as "more than", "less than", the enumeration of objects, pre-writing and pre-reading activities which would make easier the reading of "more", "less" etc.

During the activity we suggested, the children were offered the possibility:

- To use a specialized mathematical terminology as far as capacity is concerned, for example, "takes more" and "it takes less", as well as to match the cards with the containers.
- To develop the mathematical justification, as they are asked to predict, what will happen if we empty the content of one container into another one, to monitor and justify the correctness of their prediction.
- To be introduced to the concept of measurement and the use of the measuring processes.

Of course, we cannot claim that the learning result stressed here composes a permanent cognitive acquisition for the children and it probably has a limited range, as well as being related to the specific form of our activity. This fact underlines the need to use some extra evaluation methods of the didactic efforts. Moreover, the acquisition of the concept of volume "is more varied in its aspects (…) and requires more practical experience over a long period of time [Bell, et. al. 1975, p. 114].

In conclusion, what we consider didactically feasible is, besides the comparisons and the evaluations of sizes the Greek Kindergarten curriculum provides, to move further to the comparison of three-dimensional objects, aiming at the familiarity of children in this educational grade with the concepts of space and its properties.

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