Supporting mathematical argumentation of pre-school children

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ABSTRACT

In this theoretical paper, we focus on the mathematical argumentation of pre-school children about simple mathematics problems, considering the types of arguments employed and the role of the teacher in promoting or demoting certain argumentation characteristics, thus effectively establishing the mathematical argumentation norm. We propose a model synthesising the full version of Toulmin's argumentation scheme and a classification of the source of authority upon which the argumentation draws to identify the diverse ways that the teacher communicates the acceptable aspects of a mathematical argument. An exemplar application of the model to the collective argumentation of a class of four-year old children is presented.

KEYWORDS

Preschool mathematics education, argumentation, Toulmin, sociomathematical norms

RÉSUMÉ

Dans cet article théorique, nous nous concentrons sur l'argumentation mathématique des enfants d'âge préscolaire sur des problèmes mathématiques simples, en considérant les types d'arguments utilisés et le rôle de l'enseignant par l'approbation ou pas de certaines caractéristiques de l'argumentation, en établissant par cela une norme d'argumentation mathématique. Nous proposons un modèle synthétisant la version complète du schéma d'argumentation de Toulmin et une classification des sources d'autorité auxquelles l'argumentation se situe et elle y identifie les différentes manières par lesquelles l'enseignant communique l'acceptabilité ou pas des aspects d'un argument mathématique. Nous allons

présenter un exemple d'application de ce modèle sur l'argumentation collective d'une classe d'enfants de quatre ans.

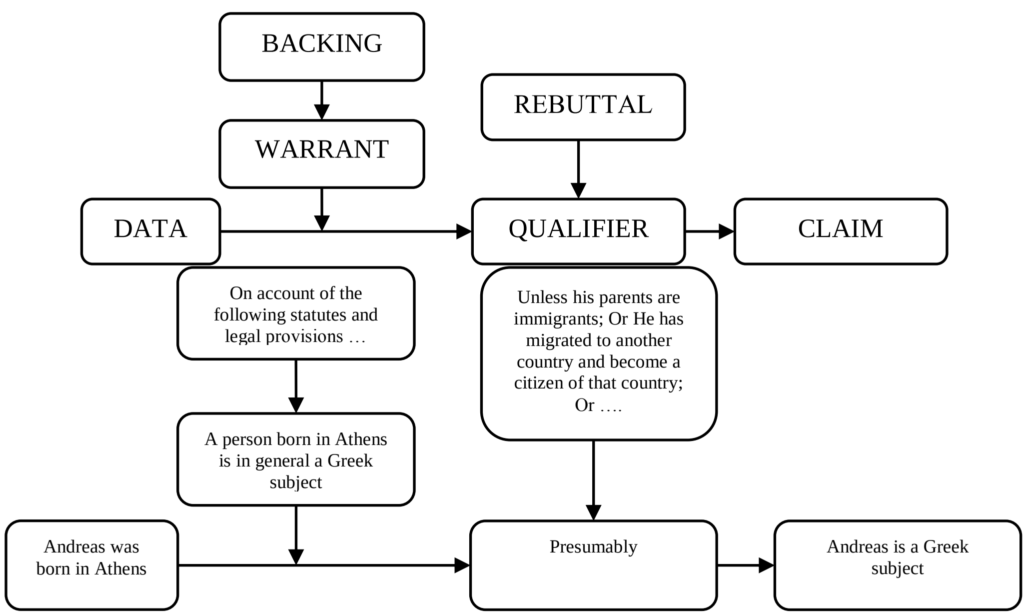
MOTS CLÉS
Enseignement préscolaire des mathématiques, argumentation, Toulmin, normes sociomathématiques

MATHEMATICAL COLLECTIVE AND INDIVIDUAL ARGUMENTATION

Echoing the fact that mathematical argumentation lies at the heart of mathematical activity, mathematics educators have investigated the mathematics argumentation, both at the individual (Inglis, Mejia-Ramos & Simpson, 2007; Weber, 2008) and at the collective level (Krummheuer, 1995; Yackel, 2002). In these investigations, various argumentation models have been employed, including Toulmin's scheme and Perelman's new rhetoric (Aberdein & Dove, 2013). In this paper, we focus on Toulmin's scheme to identify the structure of each argument.

Toulmin (1958) introduced a scheme to reveal the structure of an argument: a *Claim* is to a degree (*Qualifier*) drawn based on some *Data*, since a *Warrant* holds (on account of the support of a *Backing*), unless there is a case of a *Rebuttal*. The Data are the facts upon which the Claim (or conclusion) is based, while the Warrant is the rule, the hypothetical statement, that links this type of data (or a category of data) with the specific claim. The Qualifier indicates the degree of certainty that a Claim is drawn based upon a Claim, whereas the Rebuttal indicates instances of the bridging of the Data with the Claim is not applicable (see Figure 1).

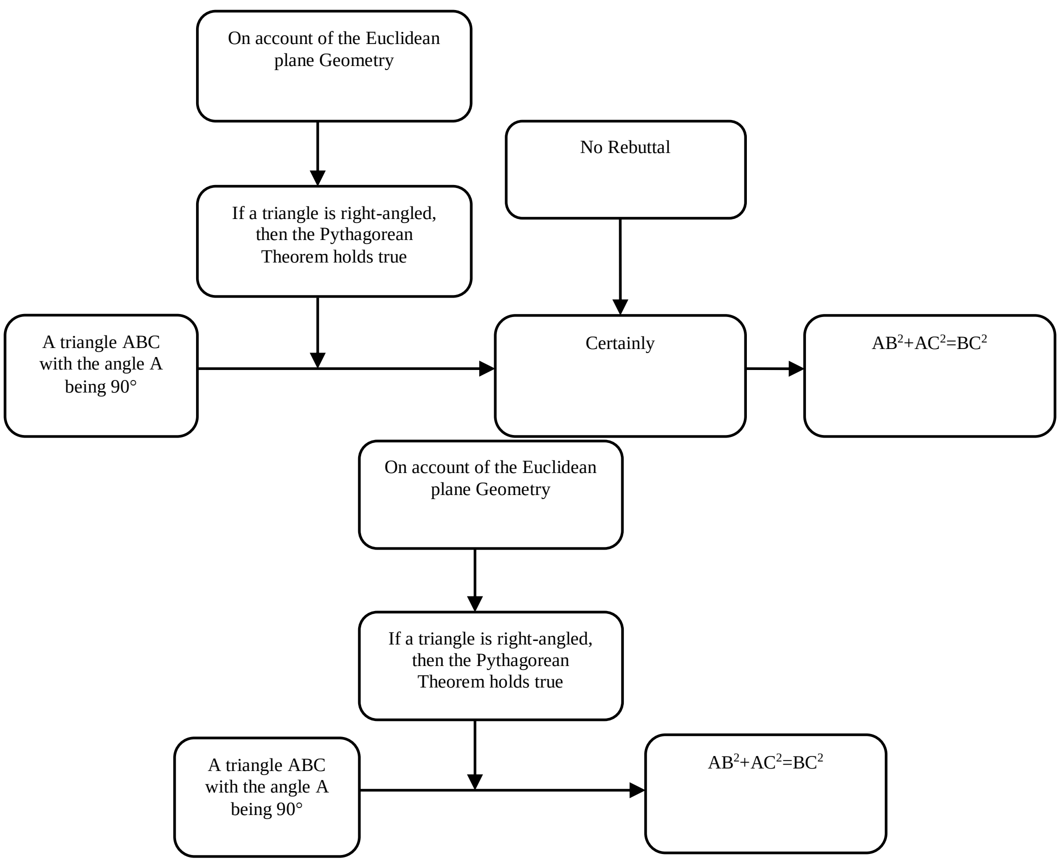
FIGURE 1



Toulmin's scheme and an example (drawing upon Toulmin, 1958)

When considering a mathematical argument, a Warrant may be the rule that "If a triangle is right-angled, then the Pythagorean Theorem holds true". Thus, given "A triangle ABC with the angle A being 90° (the Data), then "$AB^2+AC^2=BC^2$" (Claim), which is warranted by the aforementioned rule. This Warrant is supported by the implicit Backing that this rule holds true in the Euclidean plane geometry. Note that, in contrast to the Warrant being a hypothetical statement, the Backing is a categorical statement, which supports the specific applicability of a warrant to this particular argument, as well as revealing the broader system within which this warrant may be applied. Notice also, that this argument has an implicit absolute Qualifier and no Rebuttal, since this mathematical rule is characterised by the fact that it has no exceptions to the category to which is applied in the given axiomatic system and, consequently, the Claim is drawn from these Data with certainty. For this reason, researchers have argued for employing a restricted version of Toulmin's scheme (see Figure 2). In mathematics education research, Krummheuer (1995) was the first to implement a restricted version of Toulmin's scheme (Data-Warrant-Claim), though researchers have argued that individual and collective in-class argumentation may be more appropriately approached with the full version of Toulmin's scheme (for example, Inglis et al., 2007).

FIGURE 2



An example of applying Toulmin's scheme in mathematics: full (up) and restricted (below)

Considering collective argumentation, researchers have built on Toulmin's scheme to investigate the students' collective, in-class argumentation, by, for example, revealing the parallel, complementing or conflicting lines of argumentation that develop during a teaching (Knipping & Reid, 2013). Researchers linked Toulmin's scheme with other communicational aspects (Krummheuer, 1995), whilst others focussed on the teachers (Yackel, 2002). For example, Yackel (2002) utilised Krummheuer's framework to analyse the teachers' argumentation "as a means of explicating the proactive role of the teacher in inquiry mathematics classrooms and to demonstrate the broader implications of such an explication for mathematics education" (p. 439). Moreover, Conner et al. (2014) investigated teachers' supporting their students' mathematical argumentation, linking the teachers' questioning with Toulmin's argumentation scheme.

Though extensive research has been conducted with respect to mathematical argumentation, it seems that the early childhood students' mathematics argumentation has not been the focus of many research projects. In a previous work (Zacharos, Pournantzi, Moutsios-Rentzos & Shiakalli, 2016), we utilised a restricted version of Toulmin's scheme to analyse the students' reasoning and to identify the different types of argumentation they employ. In the present study, we employ the full version of Toulmin scheme to propose a model for analysing the teachers' interventions with respect to mathematical argumentation, in order to reveal their communication of the sociomathematical norms (Yackel & Cobb, 1996). Sociomathematical norms refer to the "normative aspects of mathematics discussions specific to students' mathematical activity", that is "what counts as an acceptable mathematical explanation and justification" (Yackel & Cobb, 1996, p. 461). Considering that the sociomathematical norms are founded at an early stage of the children's life, it is important to delineate the in-class argumentations practices and interactions, in order to gain deeper understanding of the children's development of practicing the 'institutional' rules of engagement to mathematical argumentation. The purpose of our approach is to reveal the links between the teachers' interventions and the students' argumentation as they develop during their in-class interactions. In this theoretical paper, first, we introduce our mode and, subsequently, we apply our model to a portion of data deriving from a different project (Shiakalli, 2013), as an exemplar.

SOCIOMATHEMATICAL NORMS: TEACHER AND STUDENTS INTERACTIONS

In this paper, we consider the level of the students' empowerment as facilitated by the teacher's actions, in the sense of the source of validation of each part of the employed mathematical argument. Thus, we investigate the argumentation sociomathematical norms (Yackel & Cobb, 1996), as communicated by the teachers with respect to the constituting elements of an argument as identified by Toulmin's scheme. At the crux of our approach lies the view of proof as a socio-temporal construction, which transcends the subjective to reach the objectified (Moutsios-Rentzos & Spyrou, 2015). Hence, the mathematical argument may be viewed to include different levels, as well as qualitatively different ways of being engaged, employing and negotiating the subjective. In line with these ideas and considering the semiotic reference of the argumentation, Moutsios-Rentzos, Spyrou and Peteinara (2014) in their work with high school students identified "two poles in the students' communication spectrum: a) *egocentric communications*, in which the individual is semiotically present within the communication […], and b) *desubjectified communications*, in which the communications focus on the phenomenon itself employing an impersonal language" (p. 42).

With respect to the noematic reference of the employed argument, Harel and Sowder (1998) identified three proof scheme types: *external conviction*, *empirical* and *analytical*. These may also be viewed as reflecting and contrasting sources of authority that validate a mathematical argument, respectively: a) a source of authority that is alien to them, b) a source of authority that refers to their own immediate and/or finite experience (perceptual or mental), and c) a source of authority that is both externally and internally referenced, in the sense that it has to be internal as it draws upon deductive reasoning and a potential experience of defined objects and accepted rules. Nevertheless, at the same time, it may be also partially externally referenced in the case that the students have not realised the epistemological status of an axiomatic system (thus accepting its ultimate, pre-existing, non-anthropological power, rather than accepting the axiomatic system as being just one of the infinite, equally valid, choices).

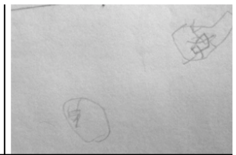
We synthesise these ideas to suggest a four-category classification of the source of authority upon which the students' mathematical argumentation draws: a) *External (EX)*, when the source of authority is the teacher, the exercise itself etc, b) *Self-referenced (SR)*, when the source of authority is the student's own construction of the situation (for example, a perceptually derived warrant or data is categorised as such), c) *Inter-subjective (IS)*, when the source of authority is self-referenced, but at the same time recognised as acceptable by a community, and d) *De-subjectified (DS)*, when the source of authority is inter-subjective, but is also acknowledged to transcend the spatio-temporal present community (for example, a written argument, valid even for individuals not present in the situation may be considered as such).

In our proposed model, we synthesise this classification with Toulmin's scheme to analyse the in-class argumentation. For example, a student may draw upon External (EX) Data to draw a Self-referenced (SR) Claim, based on an External (EX) Warrant. Moreover, the teacher's actions may differentially operate in the various elements of the argument, thus differentially empowering their students' argumentation autonomy. For example, a teacher may provide autonomy to her students with respect to the warrants they employ (by, for example, promoting De-subjectified (DS) communications), but not with respect to what they may consider as data (by promoting, for example, External (EX) communications). We posit that the proposed model, complements existing approaches (Conner et al., 2014; Yackel, 2002), facilitating the teachers to functionally reflect upon their own practices, thus supporting them in taking appropriate actions to promote their students' developing appropriate mathematical argumentation.

SUPPORTING NIKOS'S MATHEMATICAL ARGUMENTATION

In this section, we *implement the proposed mode*l to a *portion of data of another project* focusing on the mathematical problem-solving processes and resources of kindergarten children (Shiakalli, 2013). That project included a 7-month set of teaching interventions (November 2011 – May 2012) in a public kindergarten school in Cyprus. The school teacher and a researcher (also a kindergarten teacher) were present in the classroom. Twenty children participated in the study (8 girls), aged between 4 and 5 years old. In the present study, we utilise a portion of the data about a boy of that group named Nikos (pseudonym). In the beginning of the interventions, Nikos was characterised by his reluctance to be engaged in the mathematical activities and his general tendency to work on his own. The following excerpts are presented in chronological order.

FIGURE 3

Researcher: How many different ways can you detect, in order to put the five flowers in the two vases? [Nikos finds one solution 3+2, which he notes using his own notation; see right]	
Researcher: Very good. Can you think of another way? Different?	Claim – External
Nikos: There is no other way.	Claim/Warrant – Self-referenced
Researcher: Do you want to try to find another way?	Claim – External
Nikos: No. I am done. Researcher: Do you want to try it together? Nikos: I am leaving. [Nikos leaves and for a few days, he is not actively engaged to problem solving. He does not talk. He watches the other kids solving problems for a while and then he walks away doing something else]	

November 2011

Established and establishing valid Claim and Warrants: External and Self-referenced

In Episode 1 (see Figure 3), Nikos presented his answer and the Researcher intervened. According to our analysis, her intervention is an effort to legitimise the multiplicity of Claims (answers) for a given set of Data (task); the questioning implies the existence of a different Claim, which derives only from the authority of the teacher, thus being External to Nikos. Nikos argued that there was no other way, which may be warranted by an implied Self-referenced Warrant "there is only one answer". We categorise this Warrant as Self-referenced, because it may be drawn to externally set previous experiences, but it seems to be interiorised by Nikos to a degree that it can be used to doubt the implied authority of the teacher's questioning. His Self-referenced Warrant is so strong that lead him to persistently confront the researcher who urged him to look for another answer.

FIGURE 4

Researcher: With how many different ways can I put a yellow button, a red button and blue button on the belly of Little Snowy? [The children are provided with materials and a piece of paper to write down their answer]	
Teacher: [After looking on the children's work]. Many answers!	Claim – External/Self-referenced
Nikos: We found many answers.	Claim/Warrant – Self-referenced
Researcher: How many?	Claim – External
Kostas (another child): 1, 2, 3, 4, 5	Claim/Warrant – Self-referenced
Researcher: Wow! Are you done now?	
Kostas: No	Warrant – Unspecified
Nikos: There must be six. We still have one more to find.	Claim/Warrant – External

January 2012

Enriching established valid Warrants and Claims: Self-referenced and External

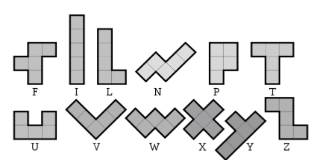
In Episode 2 (see Figure 4), the teacher again legitimises the multiplicity of Claims to a set of Data. This time her intervention was External and Self-referenced for the children, as she explicitly validated more than one of the children's answers. Nikos immediately interiorised this. Kostas (another child) answered the teachers' question drawing upon his own counting, thus producing a Self-referenced Claim warranted by his own experience. It is interesting the fact that Nikos warranted his claim that there *must* be six answers, drawing upon the fact the paper sheet had six snapshots of Little Snowy to be filled. Nikos had interiorised the sociomathematical norm that boxes should be filled with answers, which combined with his acceptance of multiplicity of answers lead him to the Externally derived answer "six". Please notice that there is also a qualitative difference in the teacher's questioning. Her questioning of "how many?" effectively enriches the qualitative characteristic "multiplicity of claims" by its quantification. In mathematics, we do not just find different answers; we need to count them. Nevertheless, such a need is still external to the students.

A few days later (Episode 3; January 2012), Nikos asked to work again with the Little Snowy task. He decided to start with a different colour, in order to implement his strategy of finding the answers. He started working. He stopped. He talked with other children and subsequently he started to work again on the problem, managing to detect all six solutions.

Nikos: 1, 2, 3, 4, 5, 6. Six solutions. Six every time with all the buttons. Different buttons, but the same answers.
Researcher: What is the important thing that you found out today? Can you explain it to me?
Nikos: It does not matter which buttons you take. The solutions are always six.
Researcher: That is, if I take four buttons, will that also lead me to find six answers?
Nikos: No! No! Colours. The buttons are three, but it does not matter which colours you take.

Nikos made a De-subjective Claim ("Six every time with all the buttons"), warranted by an (initially Self-referenced) Inter-subjective Warrant (counting and noting the answers). Moreover, his re-stating his argument to explicitly focus on the cardinality of the set of buttons, rather than on its perceptual or functional characteristics, implies that his Inter-subjective Warrant may be also De-subjectified (a different line of questioning may have helped eliciting this).

FIGURE 5

Teacher: In how many ways can you put five squares together? Each square must touch this [she shows that they must be adjacent to each other forming shape "I" (see right)] Nikos finds a solution like "P" and after a while he finds another "P" solution rotated by 90°	
Teacher: Is this a new solution?	
Nikos: Yes! I haven't found it before	Claim/Warrant: Inter-subjective
The teacher shows the "P" solution on the piece of paper with the noted solutions and she turns it to match the "new" solution.	Rebuttal: External / Self-referenced
Teacher: What do you have to say now?	
Nikos: It is the same solution, but it is turned. We should not write it down.	Rebuttal: External → Self-referenced → Intersubjective

April 2012

Establishing a valid Rebuttal: Self-referenced and Inter-subjective

In the fourth and last episode that we include in this exemplar (see Figure 5), Nikos was presented with a Rebuttal. All solutions until then were perceptually validated by the visual match with existing solutions. That time, the teacher provided an Externally-set Rebuttal: the perceptual rotation of a solution is *not* a different solution. This Rebuttal enjoys the fact that though its validity is Externally set, its recognition is Self-referenced, thus allowing its immediate interiorization by the children. This is evident in Nikos's not only including it in his verbal argumentation, but his legitimizing to be written down to the De-subjectified set of solutions.

CONCLUDING REMARKS

In this paper, we presented a model for analysing the teacher-student interaction during in-class collective argumentation, employing the full version of Toulmin's scheme combined by a classification of the utilised aspects of the argumentation that reveals the nature of the power relationship of the arguer and the argument. The purpose of our model was to map the development of the communication and the interiorisation of the sociomathematical norms with respect to mathematical argumentation. We implemented our model to a portion of data of a previously conducted project revealing the diverse ways that the sociomathematical norms are established, enriched or changed during teaching. The findings revealed the ways that the teacher's practices may affect the students' argumentation. For example, we identified the establishing of the sociomathematical norm with respect to what constitutes mathematically acceptable answer. Our ongoing research builds on these findings to further map the complex collective argumentation that occurs in the kindergarten classes with the purpose to subsequently investigate appropriate ways of supporting the teachers in their practices.

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