Peer language interactions within the mathematical problem solving process: the case of four year olds

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ABSTRACT
In this paper we aim to study the forms of verbal communication between four year old children while occupied with the mathematical problem solving process. We also look at the possibility of peer language interaction contributing to the mathematical problem solving process.

KEYWORDS
Problem solving, verbal communication, cumulative talk, exploratory talk, investigative talk

RÉSUMÉ
Dans cet article, nous nous efforçons d'étudier les formes de communication verbale entre enfants de quatre ans pendant qu'ils sont occupés avec le processus de résolution de problèmes mathématiques. Nous examinons également comment la possibilité d'interaction langagière entre pairs peut contribuer au processus de résolution de problèmes mathématiques.

MOTS-CLÉS
Résolution de problèmes, communication verbale, parole cumulative, parole exploratoire, parole investigatrice

INTRODUCTION

Young children's school experiences are considered critical for they offer the child's first contact with the school environment within which children shape the way they perceive their role as students. This is why the interest of current school programmes focuses on the elaboration and development of effective methods for teaching and learning within the school environment. (Ginsburg & Goldbeck, 2004; Clements & Sarama, 2009; Zacharos, Antonopoulos & Ravanis, 2011; Shiakalli & Zacharos, 2014).

All teaching and learning frameworks are based on the notion of effective communication (Vygotsky, 1978). Significant research interest has focused not only on teacher-student communication but on student-student communication as well. Language is believed to be used in order to convert personal reasoning into collective reasoning and practice, as well as to create personal interpretations for common experiences (Mercer, 2002). Children (as all human beings facing new situations) acquire knowledge and skills through the example and scaffolding of more knowledgeable others. At the same time learning is achieved through the gradual construction and acquisition of communal thinking skills by verbally and practically interacting with peers. Thus, members of a learning community use language in order to create their own common meanings and to achieve their own goals deriving from common interests (Vygotsky, 1978).

AIM OF THE STUDY

In this paper we aim to study the forms of verbal communication between four year old children while occupied with the mathematical problem solving process. More specifically we set out to answer the following questions:

1. Can language interaction between four year old children contribute to the mathematical problem solving process?
2. How does language interaction between four year old children facilitate the mathematical problem solving process?

THEORETICAL FRAMEWORK

Children's language interactions

Research findings suggest that group activities amongst children can offer valuable opportunities in (a) detecting solutions of a mathematical problem and, (b) constructing meaning using language as a tool. It is noted that the type of talk developed between children during their work in small groups could not be found in any other form of classroom talk (Strom et al., 2001; Mercer & Sams, 2006, Shiakalli & Zacharos, 2012).

Many researchers as, for example, Cobo and Fortuny (2000), Kramarski, Mevarech and Arami (2002) and Mercer and Sams (2006), agree in their conclusions that children's involvement in working groups does not ensure constructive cooperation. They all stress that in order for successful and constructive cooperation between members of a group to take place, it is important that children know how to cooperate with each other in order to increase the quality and the efficiency of the mathematics learning process.

The importance of the use of language as a thinking tool was studied by Teasley (1995) where thirteen year old children's quality of work was analyzed through the mathematical problem solving process. Findings suggest that talking benefited the problem solving process. Moreover the benefits of talk were even greater and more obvious in pair work. In studying the types of interaction in pairs of thirteen year olds working on the mathematical problem solving process, Kieran and Dreyfus (1998) report that the moments during which one of the two children entered the thinking sphere of the other played a crucial role in the process.

Other findings (O'Connor, 1996) show that there are negative social processes hiding behind the surface of mathematical talk which can work against group dynamics during the negotiation of meaning among the group members. It is important to note, though, that this study looked at the quality of cooperation among twelve and thirteen year olds children who had no prior experiences of group work. Sfard and Kieran (2001) found that mathematical communication among thirteen and fourteen year old children working in pairs, was notably difficult. The language used by the children lacked accuracy and clarity in what they were describing they were going to do. Moreover, the overview of social interaction between them, which did not improve throughout the study, did not seem to facilitate the children. These findings strengthen the idea that in order for group work to be constructive and successful, members of the group should comply with certain rules and follow certain norms. Moreover, in order for group work to lead to the development of cognitive and social skills, as well as to building mathematical meaning, children need to be trained in order to know how to work and interact within the team. The teacher's role is crucial since she/he has to (a) train the children in developing communication skills, (b) create groups in a way that every group member will have a positive contribution leading group work to create opportunities for the development of constructive dialogue, (c) give the children the opportunity to create a clear image of what is expected from them, and (d) encourage children to clarify and experience the elements of constructive language interaction.

Research findings (Mercer, 1995) show that teachers scarcely devote time in developing skills leading to constructive language interactions. Even when they do their instruction of how children should interact using language within group work are vague- teachers ask children to «discuss with partners" or "decide together" without having offered the children experiences of constructive interaction nor planned activities for the development of such skills (Mercer & Sams, 2006). These findings comply with Strom et al. (2001) who note that it cannot be expected by children to apply language interaction skills such as building logical arguments, critical analysis, constructive dispute, constructive argument, without prior experiences.

METHODOLOGICAL FRAMEWORK

Method

Research design was based on the principles of naturalistic research (Cohen, Manion & Morrison, 2007) while using elements from design based research (Collins, Joseph & Bielaczyc 2004). The study was based on a research team: three in-service pre-school teachers, the researcher (a pre-school teacher herself) and an observer who was the scientific director of the study. The research team designed the content of the study and assessed the teaching interventions. When and where necessary the implemented activities were improved and re-implemented.

This study formed a part of a broader educational program which extended throughout the school year. It included the implementation of educational activities aimed at the development of investigative learning practices through the mathematical problem solving process. More specifically, the program aimed at the children

- familiarizing with the mathematical problem solving process through their work on specific mathematical problems
- developing language as a thinking skill
- using language interactions during the mathematical problem solving process

The general philosophy of the study was to study children's work and interactions while working on the mathematical problem solving process at the "mathematics table" during free activity time, after they had gone through the structured teaching interventions.

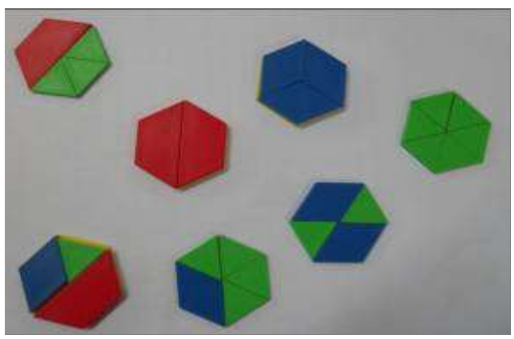
The Cyprus National Curriculum (Ministry of Education and Culture, 1996) states that from 7:45 am to 9:05 am daily, children must participate in what it calls "free activities".

During free activity time children have the opportunity to choose among a variety of activities aiming at the development of different social, cognitive and sensory skills. The Cyprus

National Curriculum calls it "free activity" time due to the fact that the activities are organized by the teacher (aiming at developing specific skills) but each child is free to choose where he/she would like to work and for how long. During free activity time in February 2012, five months after the beginning of the school year and after having completed the structured teaching interventions designed for this study, the teacher organized a "mathematics table" which consisted of a round table seating four children at a time situated in the main area of the classroom. The researcher was present at all times at the "mathematics table" observing the children. The classroom teacher, at the beginning, spent more time at the "mathematics table" offering the children help and support. This was gradually reduced and the teacher divided her time between all corners and tables organized at free activity time.

During the course of their work at the "mathematics table" children had the opportunity to work on five different mathematical problems. One of the problems was a geometry mathematical problem which consisted of creating an hexagon using other geometrical shapes (Figure 1). Children were encouraged to graphically represent their solutions on a blank piece of paper. Without prior instruction, all children chose to graphically represent their solutions by tracing the outline of the shapes (Figure 2). The problem had seven solutions and in order to successfully have solved it, children had to detect them all. This is the mathematical problem on which this paper focuses on.

FIGURE 1



Creating an hexagon using pattern blocks

FIGURE 2

Graphical representation of the solutions

The classroom teacher introduced each of the five mathematical problems to each child individually. The mathematical problems were introduced to all children in the same sequence.

After having worked on one mathematical problem several times, the teacher would introduce the following mathematical problem to the child. Thus at the "mathematics table" different children could be working on different mathematical problems at any given time. The children were not obliged to solve all problems- when the teacher introduced the following mathematical problem to a child she would ask whether he/she would like to try to solve it. If the reply was negative the teacher would introduce the following mathematical problem. After the introduction and their first time of solving each problem, the children were free to choose among the mathematical problems they had already worked on. Each child could work on a chosen problem as many times as he/she wanted and each child was free to choose whether to work on the problem alone, as a pair or in a group.

The children were not obliged to find all the solutions of the mathematical problem in one session. If they felt tired or wanted to engage in another activity, they could stop and return to the problem the following day. Each child could work on the mathematical problem on three or four consecutive days.

All problems remained at the "mathematics table" for five months (February to June, 2012).

Data Collection

Data were collected through videotaped classroom episodes, the researcher's notes while observing the children applying the problem solving process and children's answer sheets. All five problems were multiple solution problems, thus, in order to solve them the children had to graphically represent their solutions. All problems were accompanied by corresponding manipulatives as well as answer sheets.

Participants

Twenty children aged between four and five and a half years old (mean 4.8) attending a pre-school public class in Cyprus participated in the study. The researcher collaborated with the classroom teacher and observed the children's work at the "mathematics table" every morning for five months. For the purposes of this paper children are referred to with codes S1-S20 (codes were given according to the children's age ascending from S1 to S20)

DATA ANALYSIS

In order for children's verbal interactions to be analyzed Mercer's (1995; 2000) categorization was used. Thus the children's talk was distributed to the following three categories:

- Cumulative talk: participants work together in creating constant flow of a commonly understood language. Each member participates in group talk with information and in an "uncritical" and "supportive" manner. Cumulative talk is characterized by "repetitions", "confirmations" and "elaborations" (Mercer, 1995).
- Disputational talk: each participant works in order to remain diversified from the other participants. Every other participant is viewed as a threat in the materialization of every other participant's goals. During disputational talk each participants tries to convince the others with logical reasoning. Although this kind of talk can be counterproductive there are cases when it creates the circumstances under which creative thoughts arise (Mercer, 2000). Disputational talk is characterised by disagreement and individual decision

making, as well as short rotations consisting of assertions and challenges or counter assertions (Mercer, 1995).

- Exploratory talk: each participant is involved critically though constructively in the other's thoughts. Relevant information are offered for common view. Suggestions are challenged and counter-challenged only if explanations are given or alternatives are presented. The quest is for agreement amongst participants so to create a common base for progress. Knowledge and information are publically accountable and the course of thought is evident through speech (Mercer, 2002). Disputational talk, through the use of specific reasoning and validated criticism and evaluation, consists of a model dialogue during which the participants' main aim is to collectively create meaning based on structured and documented reasoning (Mercer, 2000). This is why disputational talk is considered the most fruitful type of peer talk in a classroom while at the same time it is the most difficult for young children to engage in.

"Disputational, cumulative and exploratory talk are not meant to be descriptive categories into which all observed speech can be neatly and separately coded. They are nevertheless analytic categories because they typify ways that children talk together in collaborative activities" (Mercer & Wegerif, 1999, p. 85). This typology is a useful frame of reference for understanding how talk is used by children in order to "think together" in the classroom.

The data analysis presented in this section consists of a descriptive analysis of children's talk while working on the creation of an hexagon mathematical problem during their work at the "mathematics table". Children's talk is presented and discussed in the three categories of peer talk: (a) cumulative talk, (b) disputational talk and (c) exploratory talk.

Cumulative talk

In Episodes 1, 2 and 3 all children were working on the geometry mathematical problem at the same time.

Episode 1- Counting the number of solutions
S2: I have found 1,2,3,4 .
S20: I have almost found them all. I only have one more.
S7: I have three more to find and I am finished. Then I will have found them all.

Episode 2- Process completion through all solution detection
S5: I have not finished yet. I still have one solution to find. When I find it I am finished.
S10: I will find them all. I have two more to find.
S1: I have found 1,2,3,4,5,6. One more and I am finished, I have found them all.
S4: Now I will try another idea because I also have to find all solutions. I still have three

In both above episodes cumulative talk developed among the children seemed to encourage the continuation and completion of the problem solving process. But at the same time the children's occupation with the problem solving process seemed to have encouraged the use of language among them (which in this situation fell into the sphere of cumulative talk).

Episode 3- Using ideas of another to overcome a difficulty.
S12: I have one solution left but I cannot find it.
S11: I have found them all. 1,2,3,4,5,6,7.

S12: Will you give me your answer sheet so that I can see which one I am missing?
S11: How will you know which one is missing?
S12: I will see from here (S11's answer sheet) and I will mark the ones I have found on my sheet (the child applies this idea and manages to detect the missing solution. He creates it with the manipulatives and proceeds in graphically representing it).
S12: 1,2,3,4,5,6,7. I have seven and you have seven and they are all the same. So now I am finished too.

In the above episode cumulative talk can be detected. Both children were working on the same problem individually, each following their own course. S11's recorded ideas were used as useful information by S12 in order to complete the process. Children were engaged in a productive dialogues exchanging information (dialogue quote 3.1-3.2) and explaining their thoughts (dialogue quote 3.5-3.7) leading S12 to successful completion of the problem solving process.

Disputational talk

In the following Episode two children were working on the geometry problem at the same time, individually.

Episode 4- Offering information.
S18 is working on the creation of an hexagon problem for the first time. He has been trying to create a solution using a square for some time now. S7 is working on the same problem (for the second time) sitting next to S18.
S18: Oh. . . . (turns to the teacher) can you help me here?
Before the teacher responds. . .
S7: (addressing S18) Is it a good idea to use the square to create a solution?
S18: No, I don't think so. I'll take it away. . . Oh, at last I found a new idea (takes rhombus and uses four to create an hexagon).
S7 completes the process and leaves the "math table" while S18 is still working on his problem. S6 asks to sit at the "math table" and chooses the creation of an hexagon problem for the first time. After having heard the problem by the teacher S6 takes two squares from the box.
S18: Don't try that, I tried it. Squares don't make the shape. Try another idea.
S6 places the squares back into the box and takes two rhombus.

In Episode 4, S18 began as an information receiver and then turned into an information transmitter. S18 received information, successfully used it in his own work and then successfully transmitted it to another child. It is also important that both children (S18 and S6) used the information given to them constructively avoiding to use squares in their constructions during the entire problem solving process. In the above Episode (Episode 4) children were engaged in disputational talk which was either expressed directly (episode quote 4.13) or indirectly (episode quote 4.6). It is important to note that when directly rejecting an idea, S18 offered an explanation for his comment (episode quote 4.13). Moreover, it is important to note that while S18 rejected S6's idea of using squares, he went on encouraging S6 to "try another idea" (episode quote 4.13). Both instances of disputational talk, in Episode 4, were constructive and seemed to had worked positively since both S18 and S6 managed to successfully solve the problem having used the information they had received (S18 from S7 and S6 from S18) in detecting a new solution.

Exploratory talk

In the following episode (Episode 5) two children were working as a group on a geometry problem trying to create a hexagon using other shapes. The children were working individually sitting next to each other.

> *Episode 5- Cooperation in a new solution detection.*
> S6 places a rhombus next to a triangle. S3 watches her as she works.
> *S3:* What are you going to put next?
> *S6*: I will take triangles. One blue (*rhombus)* and all the rest triangles.
> *S6*: But we've done this. Here it is *(points at the graphically represented solution).* We can use another blue one *(rhombus.)*
> *S6 takes the triangle away and places a rhombus. She then takes two triangles and completes the construction.*

The above Episode is a sample of exploratory talk: each participant was engaged critically but constructively in the process with a common goal (to solve the mathematical problem). Information was offered openly and were subjected to criticism (dialogue quote 5.2-5.3), suggestions were challenged with reason (episode quote 5.4) and alternative ideas were offered (episode quote 5.5). Agreement between both children worked a common base for progression (a new solution was detected). Throughout Episode 5 the thinking course was evident and both children's engagement in the dialogue was substantial and constructive.

RESULTS AND DISCUSSION

In this paper we looked at the possibility of young children being able to use language interactions during their engagement in the mathematical problem solving process. We studied the ways four year old children verbally interacted as well as the meaning of these verbal interactions.

Our findings suggested that language interaction among four year old children could not only be detected but had proven to be considerably elaborate and constructive. In setting to answer our research questions four main conclusions arose.

Firstly, providing guidance and experiences in using language as a thinking tool facilitated young children in using language more efficiently as a tool during the mathematical problem solving process (Episodes 4, 5). These findings were in accordance with Mercer (1995; 2000; 2002) and Strom et. al. (2001) who stressed the importance of children being trained in order to develop and acquire communication skills and skills of using language as a thinking tool. The role of the teacher proved to be crucial in acting as a model during communication skill building and using language as a thinking tool.

Secondly, during their language interactions even four year olds can develop all three types of talk as identified by Mercer (1995): cumulative talk (Episodes 1, 2, 3), exploratory talk (Episodes 4) and disputational talk (Episode 5). Language interaction among four year olds seemed to had facilitated both the continuation and successful completion of the mathematical problem solving process (Episode 3). Even in the cases where children used disputational talk this was constructive and fruitful since it let to the successful continuation of the problem solving process (Episode 4). It is also important that there were instances of exploratory talk, a type of talk considered as the most constructive type of language interaction. According to Mercer (1995;

2000) this kind of talk does not develop easily since it demands communication skills by all participants. Our data showed that even four years old children can engage in exploratory talk when (a) they have rich experiences of using language as a thinking tool, (b) they have rich experiences and have practiced communication skills and (c) the process in which they are occupied is interesting and cognitively challenging (to them).

Thirdly, the improvement of using language as a thinking tool among children seemed to had improved their individual construction of mathematical meaning (Episode 4), as Strom et al. (2001) and Mercer and Sams (2006) had reported in their work. The fact that during the structured implementation of mathematical activities the teacher systematically encouraged the use of language as a thinking tool as well as language interaction between all children, and the fact that the children had the opportunity to interact using language during their work at the “mathematics table” were elements that seemed to had contributed in the improvement of the quality of language used by the children, as well as the construction of mathematical meaning on a personal and collective level. This conclusion is in agreement with other studies presented at the beginning of our paper (Cobo & Fortuny, 2000; Kramarski, Mevarech & Arami, 2002)

Fourthly, extensive and systematic occupation of young children with the mathematical problem solving process seemed to have a positive effect on the development of language interactions. Our data showed that language interaction and the occupation with the mathematical problem solving process were interrelated in a bidirectional relationship: when language interactions were used they complemented the mathematical problem solving process (as Teasley, 1995 suggested), but also, through the systematic and extensive occupation with the mathematical problem solving process had a positive effect on language interactions among children.

In conclusion we could argue that during their work at the “mathematics table” children demonstrated their ability to engage in constructive language interactions with peers in order to (a) create working groups during the mathematical problem solving process (Episode 5), (b) constructively cooperate with another child in order to solve the mathematical problem (Episode 3), (c) use a more knowledgeable other in order to overcome difficulties during the mathematical problem solving process (Episode 4).

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