Center for Open Access in Science ▪ https://www.centerprode.com/ojer.html
Open Journal for Educational Research, 2018, *2*(1), 1-18.
ISSN (Online) 2560-5313 ▪ DOI: 00.00000/ojer.2018.00000a

An Analysis of School Mathematics Textbooks in Terms of Their Pedagogical Orientation

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Received 12 February 2018 ▪ Revised 15 May 2018 ▪ Accepted 30 May 2018

Abstract

This study aims to analyse the content of school Mathematics textbooks, based on whether they are cross-thematic in character, or not. In other words, it examines the connection of the mathematics content to knowledges drawn from other school subjects, something that is promoted as much by the curricula of Greek compulsory education, as by the institutional agencies of the European Union, in their attempt to formulate community education policy for the shaping of school knowledge. More specifically, in this research school mathematics textbooks from two consecutive Greek school grades, in which the pupils are between 12 and 13 years old approximately, and which are linked to the transition from primary to secondary education, are examined. The content analysis method was used for the approach to the research material. The results of this study revealed that Greek school textbooks do, to a small extent, achieve the pedagogical goal of a cross-thematic approach to knowledge, linking mathematics to other subjects on the curricula of Greek compulsory education.

Keywords: mathematics education, mathematics textbooks, cross-thematic approach.

1. Introduction: The importance of school mathematics textbooks in teaching

School textbooks are fundamental in the shaping of the pedagogical framework of the teachers and the pupils (Lebrun et al., 2002). In their content, the recontextualization of the scientific knowledge of Mathematics takes place and this is adapted to the goals and pursuits of the official curriculum for pupils at particular educational levels and in particular school years, and is transformed into knowledge of the school science of Mathematics (Bernstein 2000; Morgan, Tsatsaroni & Lerman, 2002). Hence, school textbooks appear as "tools" for the pedagogical guidance of the teachers, for the shaping of their teaching and the promotion of the "teaching-learning" process in such a way as to be adapted to the age-related capabilities of the pupils in the particular grades they are designed for.

Basil Bernstein's contribution to the theory for the analysis of the way in which the content of school textbooks is shaped, is significant, and our study makes especial use of the

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concept of classification (Bernstein, 1991: 2000). "Classification" refers to the way in which the contents of the curricula, and hence of school textbooks, are correlated. When the knowledge in the content of school mathematics textbooks comes strictly and exclusively from the scientific field of mathematics, then classification is strong. This is because during the creation of school mathematics knowledge, clear and distinct boundaries were implemented which make it clear that the content of the particular textbooks maintain "mathematical purity" and stands apart from the school knowledges of other subjects on the curriculum. When school mathematics textbooks use knowledges from other curriculum subjects, so that the mathematical knowledges are more easily comprehensible and more readily linked to the social reality and experiences of the pupils, then classification is weak (Bernstein, 1991; 2000). In this case a weakening of the boundaries between the various school subjects is preferred, and this occurs as much in the case of greek curricula as in the curricula in other European countries that formulate collection type curricula (Bernstein, 1991; Koustourakis, 2007; Ross, 2000). In other words, these are curricula based on the teaching of separate subjects, in contrast to the USA where syllabuses are formed "around courses as knowledge units" (Ross, 2000: 100). The selection of cross-thematic approaches for the shaping of school knowledge which is promoted by agencies of the European Union during the 21st century is linked to the implementation of weak classifications (Cedefop, 2008; Commission of the European Communities, 2000; European Council, 2009; The European Parliament and the Council of the European Union, 2006; Koustourakis, 2007). This is because inter-disciplinary approaches and a preference for the recontextualization of school knowledge is promoted, and this should combine cognitive data drawn from various scientific areas of a curriculum collection type (Bernstein, 1991, 2000; Zacharos, Koustourakis & Papadimitriou, 2014).

School textbooks implement the intended curriculum from the official education policy, transforming the teaching objectives and guidelines that are formulated there, into teaching content, in other words into a curriculum that can be enacted in the school classroom (Valverde et al., 2002; Ball, & Feiman-Nemser, 1988). More specifically, mathematics textbooks, as supplementary teaching material, have a long history and have existed since the age of ancient Greece with Euclid's *The Elements* (Fan et al., 2013). Nevertheless, research on the contribution of mathematics textbooks to teaching and learning has only been identified in recent decades. Research shows that maths textbooks comprise the basic tool teachers use in their teaching (Schmidt et al., 1996; Roth McDuffie & Mather, 2007). They help the teachers determine the content that must be taught, they determine the pace and the timing of the teaching, recommend projects for the pupils and determine either directly or indirectly what is to be assessed (Koustourakis & Zacharos, 2011).

The intended curriculum is imprinted in the school textbooks, and transforms the teaching goals and instructions formulated in it, into teaching content, that is to say, into a curriculum that can be enacted in the classroom (Valverde et al., 2002). According to some researchers (for example, Scmidt et al., 1997: 178) school textbooks belong in the category of the potentially implemented curriculum because they contribute to the potential implementation of the mathematics curriculum and are used "as intermediaries in turning intentions into implementations". This process is a form of recontextualization of school mathematics knowledge that is taught by teachers on the micro-level of their school classroom (Thompson, Senk & Johnson, 2012).

Research findings reveal that the dependence the teachers and pupils have on the mathematics textbook is more marked than the dependence on the textbooks for other subjects on the curriculum (Fan et al., 2013). The maths textbook is often the chief material the teachers base their teaching on (Grouws et al., 2004). And while the teachers, within the framework of the relative pedagogical autonomy they possess, can modify parts of the content of the textbook or the teaching strategies recommended within it, the majority of them see it as the chief expresser of the directives of the curriculum and tend to cling to it (Baker et al., 2010). In fact, it has been noted

that the use of different maths textbooks by teachers leads to the adoption of different teaching strategies (Fan & Kaeley, 2000). As emerges from the study of the content of maths textbooks, these convey pedagogical messages, which at times encourage, and at other times discourage, the realization of the curriculum (Fan, 2013).

The marked research interest in the multidimensional role of maths textbooks has led in recent decades to a host of research in which the investigation of the content of the textbooks has been studied as an independent variable (Valverde et al., 2002). According to Fan et al. (2013) the analysis of maths textbooks contains a wide range of research interests. We now quote some indicative examples of research orientations: some research attempts to trace the pedagogical intentions of maths textbooks, their structure and the maths objects from which they draw their particular content (see: Pepin & Haggarty, 2001; Pepin et al., 2013). Other research focuses on particular areas of maths, such as for example, the concepts of stochastic mathematics (Pickle, 2012), the concept of proportion (Dole & Shield, 2008), the manner of negotiating the concept of function (Mesa, 2004), on the degree of encouragement in the development of mathematical reasoning (Stylianides, 2009; Stacey & Vincent, 2009) and the way in which the issue of problem solving is negotiated (Fan & Zhu, 2000; Sun, 2011). There is a whole host of research concerned with the sociological frameworks and cultural viewpoints which are contained within the mathematics textbooks, as well as with the investigation of beliefs that are imprinted in their content, on the nature of mathematics and the manner of the formation of mathematical knowledge (see: Dowling, 1998, 2002; Koustourakis & Zacharos, 2011; Morgan et al., 2002). Other research concerns the use of textbooks in the classroom (Remillard, 2005; Zhu & Fan, 2002). In addition we encounter research which focuses on the comparative analysis of different maths textbooks from within the same country or from different countries with the objective of determining differences and similarities (see: Fan et al., 2013; Fan & Zhu, 2007; Pepin & Haggarty, 2001; Johansson, 2003; Valverde et al., 2002). More particularly, in Valverde et al.'s (2002) comparative study of school textbooks from a number of countries, problems in incorporating the recommended reforms into the maths textbooks were highlighted. In addition, in Fan and Zhu's (2007) multinational research, which focuses on problem solving, mismatches between the pedagogical aims of the curriculum and the school textbooks were found.

The aim of this study is the analysis of the content of mathematics textbooks for the sixth grade of Primary School and the first grade of Greek Junior High School (1), which were introduced in 2006 and 2007 respectively (2). The analysis criterion was the compatibility of the textbooks in question with the fundamental pedagogical principle, which is formulated in the contemporary cross-thematic curriculum of compulsory Greek education, which requires that the individual subjects of school science, such as Mathematics, should not be taught in isolation but that their syllabus should be linked to aspects of knowledge from other subjects on the curriculum (Ministry of National Education and Religious Affairs, 2003).

The cross-thematic approach to knowledge can often be found in educational reforms of the curricula in Europe, which is why the issue of the cross-thematic approach to school knowledge attracts the theoretical and research interest of a plethora of scientific papers (see: Boyle & Bragg, 2008; Harris & Grenfell, 2004; Oates, 2001; Pepin, Gueudet & Trouche, 2013; Reid & Scott, 2005; Ross, 2000; Whitty, Rowe & Aggleton, 1994). This paper endeavours to contribute to the international bibliography on the cross-thematic approach to school knowledge as it focuses on the way it is implemented in the case of Greece, through the subject of mathematics, which possesses high status on the curricula of Greek compulsory education (Koustourakis & Zacharos, 2011).

2. The cross-thematic framework

The school textbooks that are studied here set out the pedagogic principle of the reform of the curricula of Greek compulsory education at the beginning of the 21st century. The particular changes appear as the implementation of European Union decisions and it is believed that they contribute to the modernization and Europeanization of the content of Greek education (Alahiotis & Karatzia, 2006; Koustourakis, 2007; Koustourakis & Zacharos, 2011).

One outcome of contemporary educational reform was the publication of a new curriculum for Greek compulsory education (Primary Education and Junior High School) which bore the title cross-thematic approach or cross curriculum approach (Ministry of National Education and Religious Affairs, 2003) and led to the reshaping of the mathematics curricula too.

According to the Greek cross-thematic curriculum framework, the "cross-thematic approach" to school knowledge, which concerns the structuring of the content of the subjects to be taught, based on a horizontal and vertical distribution of the material to be taught, is sought. The horizontal dimension is related to the interlinking of the subject matter of the subjects that are taught in a class. The aim of the Greek cross-thematic approach is to "enable pupils to acquire a unified body of knowledge and skills, following a holistic approach to knowledge. This approach will allow them to form their own personal opinions on scientific issues that are closely interrelated and are also related with issues of everyday life" (Ministry of National Education and Religious Affairs, 2003: 18). For example, within the framework of cross-thematic teaching, during the teaching of mathematics, the linking of aspects of mathematical knowledge with knowledges from other areas of school science, such as science, history, etc., is proposed. According to Bernstein (2000), the aforementioned pedagogical orientation promotes a "weak classification", in other words the weakening of the boundaries between the different subjects on the curriculum.

The vertical dimension is related to the smooth flow of knowledge from unit to unit and from class to class. We could claim that the cross-thematic approach is comprised of two components: the way school knowledge is organised, and the teaching approach to that knowledge (Ministry of National Education and Religious Affairs, 2003; Alahiotis & Karatzia-Stavlioti, 2006).

In conclusion, what is aimed at through the cross-thematic approach is the horizontal connection of school objects to fundamental concepts that are encountered in a number of school subjects within the same grade, and often their vertical connection with school subjects from different classes.

We should note that cross-thematic learning and teaching approaches comprise a contemporary pedagogical issue that places emphasis on cross-thematic approaches in learning and teaching. For example, according to Bjorklund, & Ahlskog-Bjorkman (2017: 99) "children need to be offered a relevance structure for their exploration of different phenomena. This relevance structure supports meaning-making, in that earlier experiences and present resources are joined together, which offers an opportunity to experience the phenomena in ways, not previously possible". The cross-thematic approach integrates different areas, such as mathematics, science, geography, history, literacy, etc., something which provides the pupils with the opportunity to delve more deeply into the issues they are dealing with and to approach them in new ways.

Based on the aforementioned, for the purpose of the present research paper, we regarded the cross-thematic approach as the intentional pedagogical attempt to link Mathematics with other scientific areas of school knowledge that are taught in each school year. Based on this general pedagogical principle, in the school years 2006-2007 and 2007-2008 new school Mathematics textbooks for Primary and Secondary Education respectively were published, which are still in use today.

3. The research questions

In this paper we concern ourselves with the intended curriculum, as this is set out in the reformed curriculum and is particularised in the school textbooks, and not with the implemented curriculum, in other words the actual teaching practices that teachers develop in their classroom. As was mentioned previously, the aim of the present research is to investigate the extent to which the school textbooks that are used in the last year of Primary Education and the first year of Secondary Education set out the pedagogical framework of the cross-thematic approach.

More specifically, the research questions that we will attempt to investigate are the following:

- Are the pedagogical principles of the cross-thematic approach set out in the content of the Mathematics textbooks for the final year of Primary Education (sixth year of Primary School) and the first year of Secondary Education (first year of Junior High School)?
- Are there any differentiations of a qualitative or quantitative character in the content of the examined textbooks in terms of the cross-thematic approach? Here, two dimensions attracted our research interest: the potential differentiations during the transition from primary to secondary education, as well as the differentiations within each series of textbooks.

4. Methodology

4.1 *Material for the collection of empirical data*

The material for the collection of our research data was the official Mathematics teaching material used by students in the two school grades being examined. This material, in the case of the last year of Primary Education consists of the student's textbook and four workbooks (volumes a,b,c, and d), while for the students in the first year of Secondary Education, there is only the student's textbook. For the analysis of the mathematics texts in question, the most recent editions of the books were used (Kassoti et al., 2014a, 2014b; Vandoulakis et al., 2012).

Our criteria for the choice of the particular textbooks was the fact that although they belong to different levels of education (Primary and Secondary), the school years to which they correspond are consecutive in the Greek education system and, in addition, much of the mathematical content that they contain is common to both years. Consequently, the particular choice affords the opportunity to investigate the second research question which investigates aspects of the transition from primary to secondary education and the highlighting of potential "gaps" between the two grades. This is because research on the transition from primary to secondary education often highlights a number of important changes in the learning environment between the two grades and this move from one school grade to the next resembles a move to a different "world" (Darragh, 2013).

4.2 *Structure of the school textbooks*

The primary school textbook has the following structure: It is divided into "chapters", where each chapter corresponds to one teaching unit of two pages. For example, there is one chapter on natural numbers, another chapter on decimals and so on. Each chapter contains two "activities", which are carried out in class and two "applications" which are usually solved exercises which are dealt with either in the class or at home by the pupils. Finally, there is a unit entitled "Questions for self-assessment and discussion" which contains evaluation questions. The four volumes of workbooks contain exercises and "extension activities" which correspond to the

"chapters" in the textbook. It should be noted that it is in the "extension exercises" where the greatest number of cross-thematic activities are to be found.

Finally, the teaching units of the secondary school mathematics textbook contain two or three "activities" which are sometimes formal exercises and other times require a more investigative approach from the pupils. The activities are followed by a generalization with the setting out of rules and finally there are examples of solved exercises.

4.3 *Unit of analysis*

The criteria, according to which the content analysis of the Mathematics textbooks was carried out, was their cross-thematic content. The school textbooks were analysed using the content analysis method taking the "sentence" as unit of analysis (Morais et al., 1999; Koustourakis & Zacharos, 2011; Zacharos, Koustourakis & Papadimitriou, 2014). Here the concept of "sentence" is not understood in terms of its grammatical content, but rather its semantic content. Hence, the sentence may be made up of an extract from a text which describes a complete teaching activity, which has a particular teaching objective. Consequently, the sentence can be a theory text, an activity for the students, a maths application, a maths exercise or a graph in the cases where this constitutes an autonomous teaching object.

For example, in Figure 1 in the same activity there are three autonomous units of analysis. The first requires the children to colour in up to the point where the container fills up, the second requires them to match decimal numbers and fractions on a number line, while the third requires the formulating of a rule for converting decimal numbers into fractions.

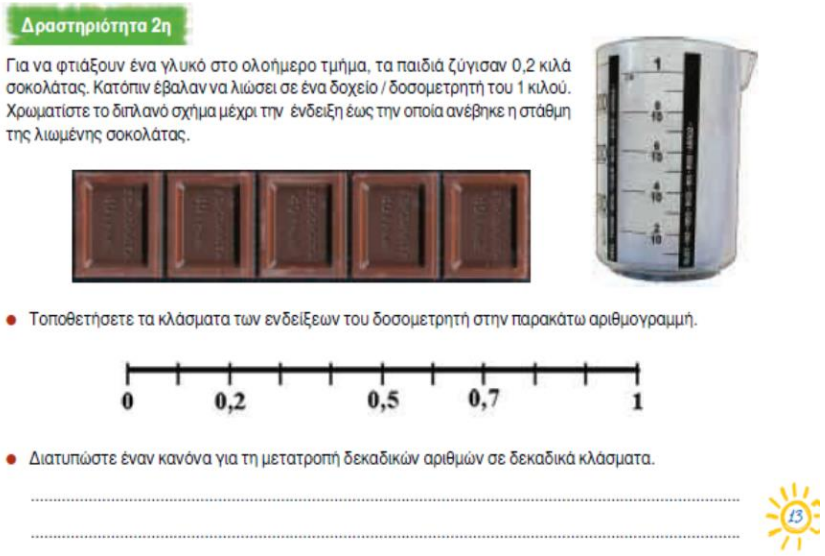


Figure 1. Three different "units" in the same activity (Kassoti et al., 2014b, student's textbook, p. 13)

The activity mentions the following:

Activity 2

The children weighed 0.2 kilos of chocolate in order to make a sweet at school. They then melted it in a 1 kilo container/measuring jug. Colour the jug on the right up to the point reached by the melted chocolate.

- Place the fractions in the markings on the jug on the number line below.
- Formulate a rule for converting decimal numbers into fractions.

Then, the units of analysis were classified based on the following criteria:

The first criterion for classification was whether the sentence was cross-thematic, or not. Based on this criterion, the sentences – units were classified into those where the content was cross-thematic and those where it wasn't. In the case of the sentences that were classified as cross-thematic, the scientific object (or objects) involved in the sentences in question was determined.

The second criterion for classification was the mathematics textbook which contains the sentence in question. As has already been mentioned for Primary education we had the student's textbook and the workbooks a, b, c, and d, while for Secondary education, the student's textbook.

The third criterion was the thematic unit in which the content of the sentence was integrated. Here we distinguished the units: Numbers & Algebra, Geometry & Measurement, Stochastic Mathematics and Statistics.

Finally, the fourth criterion was the teaching position the chosen sentence occupies in the textbook. More specifically, according to the structure of the textbooks, the sentences can be integrated into the category of introductory activities, the section on theory and additional texts, the examples and applications, in the exercises and consolidation exercises.

Based on the previous criteria, the data was processed using the statistical software SPSS (version 22).

4.4 *Indicative examples of the sentence classification*

The first example was drawn from the textbook for the final year of Primary Education (see Figure 2). This particular extract constitutes a "sentence", based on the definition we gave, since it constitutes an autonomous activity with a specific target. In addition, it could be characterised as cross-thematic because it is included in the teaching unit entitled "addition and subtraction" and apart from the inverse operations of addition and subtraction, provides an opportunity to make mention of concepts from physics as well as social concepts.

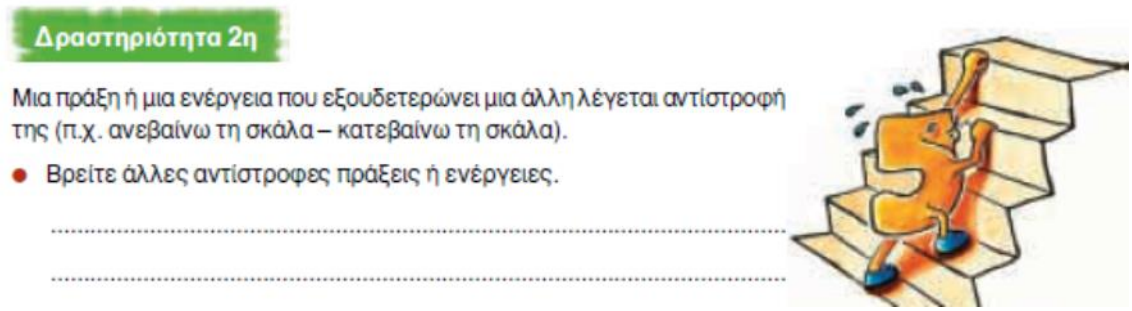


Figure 2. Example of a cross thematic sentence (Kassoti et al., 2014b, textbook, p. 17)

The activity-sentence mentions the following:

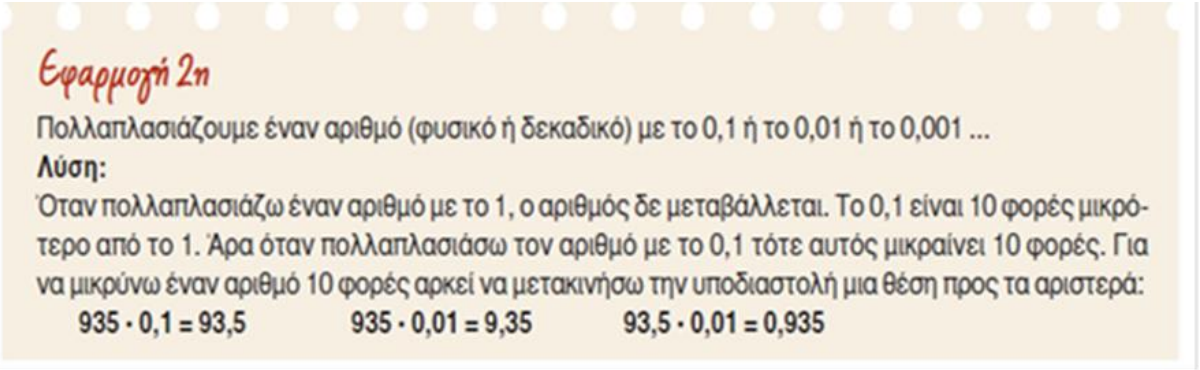
Activity 2

An act or an operation that cancels out another is called its inverse (e.g. I ascend the stairs – I descend the stairs).

- Find some other inverse operations.

The second example is a sentence from the student's textbook, also from the final year of Primary Education (see Figure 3). This sentence is characterised as non-cross-thematic, as it

has a purely mathematical content, which is integrated into the unit "Numbers – Algebra". It is to be found in the textbook unit entitled "Solved examples and applications", since it is an application concerning the execution of sums between natural and rational numbers.



Εφαρμογή 2η

Πολλαπλασιάζουμε έναν αριθμό (φυσικό ή δεκαδικό) με το 0,1 ή το 0,01 ή το 0,001 ...

Λύση:

Όταν πολλαπλασιάζω έναν αριθμό με το 1, ο αριθμός δε μεταβάλλεται. Το 0,1 είναι 10 φορές μικρότερο από το 1. Άρα όταν πολλαπλασιάσω τον αριθμό με το 0,1 τότε αυτός μικραίνει 10 φορές. Για να μικρύνω έναν αριθμό 10 φορές αρκεί να μετακινήσω την υποδιαστολή μια θέση προς τα αριστερά:

$935 \cdot 0{,}1 = 93{,}5$ $935 \cdot 0{,}01 = 9{,}35$ $93{,}5 \cdot 0{,}01 = 0{,}935$

Figure 3. Example of a sentence with non-cross thematic content (Kassoti, et al., 2014a, p. 20)

The activity-sentence mentions the following:

2nd application

We multiply a number (natural or decimal) by 0.1 or 0.01 or 0.001 ...

Solution:

When I multiply a number by 1, the numbers remains the same. 0.1 is 10 times smaller than 1. So when I multiply the number by 0.1, then it becomes 10 times smaller. To make a number 10 times smaller I just need to move the decimal point one place to the left:

935 x 0.1 = 93.5 935 x 0.01 = 9.35 93.5 x 0.01 = 0.935)

In order to ensure reliability in the classification of the sentences, the sentences were classified by each of the four authors of the present research paper, working autonomously. The comparison of the final classifications by the four reviewers showed a convergence in at least 75% of the cases in each classification (Morais et al., 1999; Koustourakis & Zacharos, 2011). In cases where a sentence didn't present an acceptable convergence, it was omitted and not included in the material for this research.

5. Results

The presentation of the results from our study includes qualitative and quantitative investigation. In the qualitative investigation the framework for the recognition and classification of the sentences-units was used, which is described analytically in the section on methodology. For the quantitative analysis and the brief presentation of the research data, contingency tables were used. Finally, to check for differences in the two independent samples (school textbooks from Primary and Secondary Education) which is linked to the second research question, the statistical test chi-square was used.

5.1 *Concerning the cross-thematic criterion*

From the total number of 1860 mathematics sentences that were recorded, 1630 (87.63%) are not of a cross-thematic nature and 230 sentences (12.37%) are cross-thematic. More analytically, by school year (see Table 1): In the textbooks for year 6 of Primary School, a total of 946 sentences were recorded. Of these, 168 (17.76% of the total of the sentences for this class) have

cross-thematic content, while the remaining 778 (82.24%) are purely mathematical without cross-thematic characteristics. In the textbook for the first year of Secondary School, from a total of 914 sentences 62 (6.78%) are of a cross-thematic character, while the remaining 852 (93.22%) are purely mathematical.

Table 1. Frequencies of sentences in terms of their cross thematic content

	Non-cross thematic sentences	%	Cross thematic sentences	%	Total number of sentences	%
6th of Primary School	778	82.24	168	17.76	946	100.00
1st of Junior High School	852	93.22	62	6.78	914	100.00
Total	1630	87.63	230	12.37	1860	100.00

The data in Table 1 lead to the following conclusions: Despite the clearly stated intention of the curriculum for the teaching of mathematics within the framework of a cross-thematic teaching approach, the school textbooks meets this requirement to only a very small degree. In contrast, their authorship follows the conventional form of writing school textbooks where the content is purely mathematical. Consequently, in the content of the school textbooks examined, strong classification (Bernstein, 1991, 2000) of mathematical school knowledge predominates. A second finding is that the textbook for Mathematics in year six of Primary School is more adapted to the cross-thematic perspective than the textbook for the first year of Junior High School. More analytically, 17.7% of the sentences in year 6 of Primary School have a cross-thematic character, as opposed to 6.78% of sentences for the first year of Junior High School. This differentiation is statistically significant (x^2 (1, N=1860)=51.676, p<0.001). A further analysis of the school textbooks for year six of Primary School (Student's Book and Workbooks) revealed differentiations which are recorded in Table 2. More precisely, the workbooks embrace the cross-thematic spirit to a greater degree than the Student's Book. In fact, the differentiation is statistically significant (x^2 (4, N=946)=30.100, p<0.001).

Table 2. Frequencies of sentences in terms of their cross thematic content in the printed material for the sixth year of Primary School

Type of printed material	Non-cross thematic sentences	%	Cross thematic sentences	%	Total	%
Student's Book	437	86.71	67	13.29	504	100.00
Workbook-a	101	80.16	25	19.84	126	100.00
Workbook-b	96	79.34	25	20.66	121	100.00
Workbook-c	87	82.86	18	17.14	105	100.00
Workbook-d	57	63.33	33	36.67	90	100.00
Total	778	82.24	168	17.76	946	100.00

5.2 *Scientific areas involved in the cross-thematic mathematical sentences*

Regarding the scientific areas involved in the cross-thematic mathematical sentences, the following points were noted (see: Table 3):

In school textbooks for year 6 of Primary School, of the 168 mathematical sentences that are cross-thematic, 28 (16.67% of the cross-thematic sentences) are linked to issues from Geography, 11 (6.55%) to issues from Language, 46 (27.38%) from Art, 25 (14.88%) to issues from History, 11 (6.55%) to the subject Social and Political Education (SPE), 10 (5.95%) to Physical

Education and 37 (22.02%) to the subject of Physics. Setting the contribution of the scientific fields in cross-thematic mathematics sentences hierarchically, we see that the first place is occupied by the subject of Art followed by Physics.

In the school textbook for the first year of Gymnasion, of the 62 cross-thematic mathematics sentences, 6 (9.68% of the total number of cross-thematic sentences) were related to the subject of Geography, 6 (9.68%) to the subject of Language, 17 (27.42%) to Art, 25 (40.32%) to History, 1 (1.61%) to Music, 6 (9.68%) to Physics and 1 (a percentage of 1.61%) to Biology.

Table 3. Scientific objects which appear in the cross thematic mathematical sentences in the textbooks for the sixth year of Primary School and the first year of Junior High School

Subject	Sixth Year of Primary School		First year of Junior High School		Total	
	Non-cross thematic sentences	%	Cross thematic sentences	%	Total	%
Geography	28	16.67	6	9.68	34	14.78
Language	11	6.55	6	9.68	17	7.39
Art	46	27.38	17	27.42	63	27.39
History	25	14.88	25	40.32	50	21.74
Social & Political Education	11	6.55	*	*	11	4.78
Music	*	*	1	1.61	1	0.43
Physical Education	10	5.95	0	0.00	10	4.35
Physics	37	22.02	6	9.68	43	18.70
Biology	*	*	1	1.61	1	0.43
Total	168	100.00	62	100.00	230	99.99

Note: The fields marked with an asterisk (*) denote that the corresponding subject is not taught.

It should be noted that only seven mathematics sentences (three from the textbook for the first year of Junior High School and four from the final year of Primary School) are linked to more than one scientific object. To be precise, in one mathematical sentence, issues from Physics and History are involved, in one issues from Language and Physics, in one issues from Geography and Social & Political Education, in one issues from Physics and Art, in one issues from History and Music, in one issues from Physics and Biology and finally in another, issues from History and Art.

5.3 *Cross-thematic sentences by themed units*

Regarding the cross-thematic nature of sentences by themed unit in the content of the textbooks under examination, the following was ascertained:

In the printed material for year 6 of Primary School (see Table 4), in the themed unit "Numbers-Algebra", out of a total of 651 sentences, 95 (14.59%) were cross-thematic, in the unit "Geometry-Measuring" out of a total of 252, 60 (23.81%) were cross-thematic and in the unit "Stochastic Mathematics-Statistics", out of a total of 42 sentences, 13 (30.23%) were cross-thematic. Here the qualitative findings show that the themed unit "Stochastic Mathematics – Statistics" is the one with the most strongly cross-thematic character, in fact with a statistically significant difference as against the other units ($x^2(2, N=43)=15.365, p<0.001$).

Table 4. Frequencies of sentences in terms of their cross thematic content in the printed material for year 6 of Primary School, by themed unit

Themed Unit	Non-cross thematic sentences	%	Cross thematic sentences	%	Total	%
Numbers-Algebra	556	85.41	95	14.59	651	100.00
Geometry-Measuring	192	76.19	60	23.81	252	100.00
Stochastic Mathematics-Statistics	30	69.77	13	30.23	43	100.00

In the student's book for the first year of Junior High School, the corresponding findings are recorded in Table 5. Here, as we mentioned previously, there are few cross-thematic sentences out of the total number of sentences, with the themed unit "Geometry-Measuring" displaying the strongest cross-thematic character. Nevertheless, the difference in the cross-thematic character of this unit as against the other themed units is not statistically significant (x^2 (1, N=914)=2.300, p>0.05). This is because in the content of the textbook for the first year of Junior High School, strong classification of school mathematical knowledge predominates (Bernstein, 2000).

Table 5. Frequencies of sentences in terms of their cross thematic content in the student's book for the first year of Junior High School, by themed unit

Themed Unit	Non-cross thematic sentences	%	Cross thematic sentences	%	Total	%
Numbers-Algebra	549	94.17	34	5.83	583	100.00
Geometry-Measuring	303	91.54	28	8.46	331	100.00
Stochastic Mathematics-Statistics	*	*	*	*	*	*
Total	852	93.22	62	6.78	914	100.00

5.4 *Cross-thematic mathematics sentences by teaching position*

As far as the teaching position of the cross-thematic mathematics sentences is concerned, in other words the position in which the sentences are placed according to the structure of each chapter, the following was discovered: In textbooks for the first year of Primary School, we noticed that the majority of cross-thematic sentences were located in the section on "Introductory activities" (see Table 6). Indeed, this differentiation in the concentration of sentences is statistically significant (x^2 (3, N=946)=41.820, p<0.001).

Table 6. Positioning of sentences in textbooks for the sixth year of Primary School

Teaching position	Non-cross thematic sentences	%	Cross thematic sentences	%	Total	%
Introductory activity on mathematical concepts	90	65.22	48	34.78	138	100.00
Theory and supplementary texts	77	90.59	8	9.41	85	100.00
Solved examples and applications	113	93.39	8	6.61	121	100.00
Exercises and practice activities	498	82.72	104	17.28	602	100.00
Total	778	82.24	168	17.76	946	100.00

In the school textbook for the first year of Junior High School, the majority of cross-thematic sentences are to be found in the section of supplementary texts (see Table 7). And in this case the differentiation in the concentration of cross-thematic sentences is statistically significant (x^2 (3, N=914)=33.800, p<0.001).

Table 7. positioning of sentences in textbooks for the first year of Junior High School

Teaching position	Non-cross thematic	%	Cross thematic	%	Total	%
Introductory activity on Mathematical concepts	89	93.68	6	6.32	95	100.00
Theory & additional texts	122	82.43	26	17.57	148	100.00
Solved examples & applications	155	93.94	10	6.06	165	100.00
Exercises and practice activities	486	96.05	20	3.95	506	100.00
Total	852	93.22	62	6.78	914	100.00

6. Conclusion and discussion

In this research, content analysis of Greek Primary and Secondary Education school mathematics textbooks was attempted. More precisely, we analysed school mathematics textbooks for the final year of Primary Education (year 6 of Primary School) and the first year of Secondary Education (year 1 of Junior High School). Our research interest focused on the degree of compatibility of the pedagogical content of school textbooks with the objectives of the curriculum for the teaching of mathematics, which sets the cross-thematic approach as a central direction for teaching. Consequently, the criterion for the analysis of mathematics texts was the extent to which they set out the principles of the cross-thematic approach, contributing to the weakening of the boundaries between the subjects of the curriculum and linking school mathematics knowledge to knowledge of specific subjects.

Regarding the first research question, the findings show that the school textbooks, which were the object of the research, are written in a rather conventional way, as the majority of the sentences-units of analysis which were recorded (87.63% of the total number of sentences) are not of a cross-thematic character. In other words, the school textbooks in question mainly present a purely mathematical content (according to Bernstein "strong classification") and only partially involve topics from other subjects in their material, despite the fact that the declared intention of the curriculum is the coupling of mathematics with other scientific fields.

In the sentences with a cross-thematic character, mathematics is mainly coupled with one subject of the curriculum for the teaching year. Very few sentences connect more than two subjects of the curriculum.

With regards to the first aspect of the second research question, that is to say, the differentiations during the transition from primary to secondary school, a differentiation (statistically significant) was found between the printed educational material of the two school years in terms of their cross-thematic character: the cross-thematic approach is more marked in the last year of Primary School (17.76% of the total number of units-sentences for the school year) than in the first year of Secondary Education (6.78% of the total number of units-sentences of the class). This fact allows us to point out a form of "inconsistence" in the transition from one school year to the next as indicated in the relevant research (e.g. Darragh, 2013). In the writing of school mathematics textbook for the first year of Junior High School, the existence of "strong classification" (Bernstein, 1996, 2000) of the scientific object of Mathematics in relation to other scientific areas, is clearly evident.

With regards to the second aspect of the research question, in other words, the differentiations within each textbook module, the unit of school mathematics for the sixth year of

Primary School where the cross-thematic approach is most apparent is the unit on Stochastic Mathematics and Statistics, while the corresponding unit for the first year of Junior High School is the unit "Geometry – Measuring". In the case of Stochastic Mathematics and Statistics the framework for the introduction and development of the concepts is drawn from examples from everyday life or from other scientific areas (like Biology).

The greatest percentage of cross-thematic activities are to be found in the introductory activities, which every teaching unit of the textbook for sixth year of Primary School contains, as compared with other sections of the textbooks. This fact probably manifests the intention of the textbooks' writers to introduce new mathematical concepts by linking them to knowledge the students possess from their experience or from the teaching of other subjects on the curriculum. These introductory activities are usually approached through constructivist teaching practices, which require investigative self-activation and cooperation on the part of the students. However, in order to be able to carry out open activities of this type, sufficient teaching time is required, time which is not available due to the stiflingly intense teaching pace of the mass of the curriculum. This is the reason why teachers in Primary Education often treat these particular activities with a conventional teacher-centred manner of approach, which reduces the amount of time students are required to spend on the activity. We should note that in Valverde et al.'s (2002: 67) comparative study of school mathematics textbooks from a number of countries, in the few cases of textbooks that incorporated "multiple topic themes" these are introductory and then "the rest of the book progresses with a succession of single-topic themes".

In school mathematics textbook for the first year of Junior High School, most cross-thematic sentences are located mainly in inserted texts (in boxes) with historical information on mathematical concepts which are to be taught. The usual practice of teachers, who teach mathematics in the first year of Junior High School, is to encourage the students to read the information in the boxes at home, alone, as these rarely constitute the object of a collective teaching approach in class.

In school mathematics textbook for the first year of Junior High School, most cross-thematic sentences are located mainly in inserted texts (in boxes) with historical information on mathematical concepts which are to be taught. Such is the case in Figure 4, which is to be found in the unit entitled "Natural numbers". The usual practice of teachers who teach mathematics in the first year of High School, is to encourage the students to read the information in the boxes at home, as these rarely constitute the object of a collective teaching approach in class.

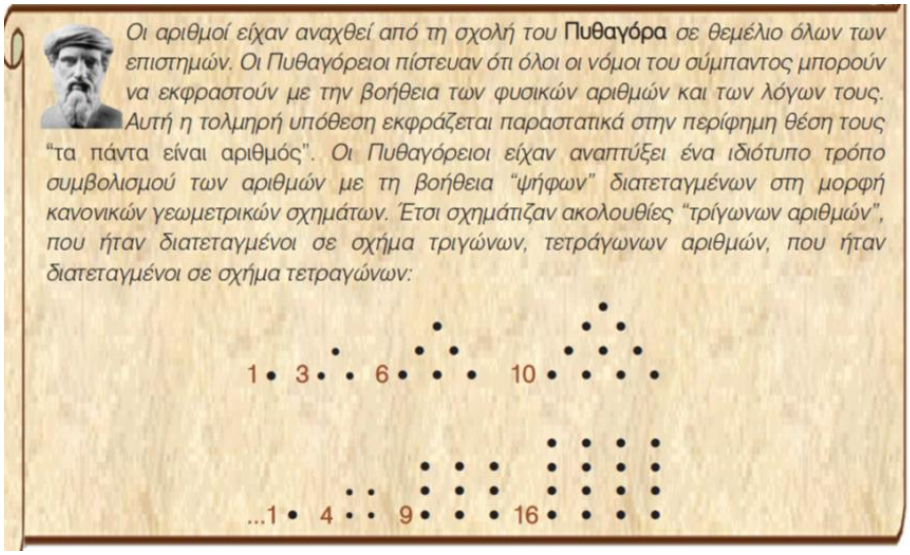


Figure 4. Example of a historical insert (Vandoulakis et al., 2012, p. 121).

Note: this particular insert refers to the school of Pythagoras and the formation of "triangular" and "square" numbers.

In conclusion, as is highlighted in Valverde et al.'s (2002: 161) research too, the proposed reforms are often not widely incorporated into the school textbooks. In addition, the answer to how "they bridge the chasm between the intentions of curricular policies and the realities of classroom implementations – is no simple matter".

Indeed, as the findings of the present research show, the transition process from the field of policy design and the implementation of pedagogical approaches, to the creation of suitable educational material, in which the pedagogical principles in question are adopted, such as the school textbooks, is not a linear or self-evident process. This research shows that during the process of the reshaping of the principles of the official curriculum into the contents of mathematics school knowledge significant adaptations and "distortions" take place. This fact points to the existence of inherent weaknesses in the design of the curriculum and the writing of school textbooks, which often do not take into account important parameters of the educational act, such as the stifling framework which is determined for the teaching of mathematics. This framework defines in detail exactly what is to be taught and how much time is to be spent on each particular unit while it doesn't take into account the "inertia" of the school institution to assimilate the changes, which is often due to a lack of suitable preparation of those involved with education (school textbook authors, teachers and others).

Coming to the end of this paper, we would like to highlight the importance of the elaboration of comparative research on school textbooks in European countries where pedagogies that encourage a cross-thematic approach are promoted in the curricula. This is because the cross-thematic principle seems to constitute a central point for the promotion of the aim of the creation of a European community of knowledge (Commission of the European Communities, 2000; European Council, 2009; The European Parliament and the Council of the European Union, 2006).

Notes

(1) In the Greek education system, Primary Education lasts six years and the final year (ST' Dimotikou in Greek) is made up of pupils aged approximately 11-12. The first stage of Secondary Education lasts for three years and the first year of this level (A' Gymnasiou in Greek) is made up of pupils aged approximately 12-13.

(2) In the Greek educational system, textbooks are proposed by the Institute of Education, an agency under the supervision of the Ministry of Education, and are the same for all schools.

Statement

The authors declare no competing interests.

This research did not receive any specific grant from funding agencies in the public commercial, or not-for-profit sectors.

Acknowledgements

Authors would like to thank the reviewers for their useful suggestions on the manuscript.

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