

RESEARCH-BASED DESIGN OF A TEACHING AND LEARNING SEQUENCE FOR THE FIRST LAW OF THERMODYNAMICS

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In this study, we present the design principles and the respective development of a Teaching and Learning Sequence (TLS) in the field of introductory thermodynamics, with particular focus on the First Law of Thermodynamics (FLT), for upper secondary school students. The TLS design takes into account the poles of knowledge, learning and teaching that correspond to three theoretical domains; (a) epistemological aspects related to the FLT, (b) students' cognitive capacity for dealing with the various features of the FLT and (c) pedagogical approach of this topic. Regarding the epistemology of the field, we analyze the historical framework in which the FLT emerged and the textbook approaches for the introduction of this law. The cognitive analysis includes the categorization of the multi-dimensional alternative frameworks and the reasoning patterns that the students put forward in explaining phenomena related to the FLT. About the pedagogical aspects of the TLS, we analyze the constructivist approach as a learning alternative for the traditional approach, which prevails among science curricula. The conclusions drawn by the three-level analysis formulated the design principles for a medium-level TLS (12 sub-sections) that covers the FLT along with several introductory thermodynamic topics. The results from the implementation of the TLS are presented briefly.

Keywords: Science Education, Teaching Learning Sequences, Physics

INTRODUCTION

In science education contexts, the development of a Teaching and Learning Sequence (TLS) aims to students' facilitation in their pursuit of understanding the established scientific knowledge. The design of a research-based TLS relies particularly on the results of preceding studies for a chosen topic (Ruthven, Laborde, Leach, & Tiberghien, 2009). In this study, we present the design principles and the respective development of a research-based TLS that addresses the First Law of Thermodynamics (FLT) as an introductory physics thermodynamics topic.

The FLT suggests an expression of energy conservation throughout a thermodynamic system and its surroundings. It also describes the distribution of energy through the conversions of the implicated energy entities (heat, work, and change in internal energy). Therefore, it is considered a fundamental principle of thermodynamics and its understanding is crucial for all experimental scientists, technicians and modern citizens.

In this case, we design a TLS for the FLT addressing the second year of secondary school. For this educational level, the research regarding the teaching and learning of thermodynamics, beyond the understanding of basic concepts such as temperature and heat, is rather limited (Meli, Koliopoulos, Lavidas, & Papalexioiu, 2016). Our main goal is the improvement of students' representations regarding energy conservation. This design takes into account three basic pivots; knowledge, learning and teaching. Each of these pivots corresponds to a theoretical domain; these are respectively epistemology, cognition, and

pedagogy (Buty, Tiberghien, & Le Maréchal, 2004). In the following sections, we elaborate on these domains and we present a sample of the TLS as the product of this analysis.

DESIGN PRINCIPLES OF THE TEACHING AND LEARNING SEQUENCE

Epistemological analysis: historiographical and textbook analysis regarding the FLT

By the end of the 17th century, scientists had fully realized that materials at a gaseous state behave like any other fluid and conducted experiments for the investigation of the thermal properties of gases. At first, their main focus was on the interpretation of the nature of heat, which involved microscopic issues that could not touch just yet; the fundamental work of John Dalton (1766-1844) was about a century away from being established and representations of the micro-level world were unattainable (Cardwell, 1971). Although a few scientists, like Daniel Bernoulli (1700-1782) worked toward a Newtonian-like explanation for thermal properties, the absence of material points inevitably gave rise to a new branch of physics that developed aside from the dominating Newtonian system (Harman, 1982). In that sense, “physics of heat” was the first scientific revolution that emerged after Isaac Newton’s (1643-1727) era.

One additional particularity of this new physics was that its applications preceded its theoretical establishment for about a century. In 1712, Thomas Newcomen (1663-1729) constructs the first functional steam engine, which plays a substantial role in the Industrial Revolution. The urgent need for the improvement of its efficiency shifted scientists’ focus from the nature of heat to the conservation of heat. Many prestigious scientists, like Sadi Carnot (1796-1832) made a clear distinction between microscopic hypothesis and developments at the macroscopic level (Meli & Koliopoulos, 2019). Through their work, the initial statement of heat conservation evolved to the notion of heat-work equilibrium and finally to energy conservation, which was expressed with the complete form of the FLT by Rudolf Clausius (1822-1888) in 1854 (Clausius, 1854, p. 484).

The principle of energy conservation in a qualitative and quantitative form through the explicit formulation of FLT affected all scientific fields. Each discipline introduces the FLT under its particular scope and therefore diverse textbook approaches are describing the same thermodynamic law (Christiansen & Rump, 2008). Physics and engineering textbooks offer many interesting perspectives that are often in agreement and complement each other. These refer to multiple statements of the FLT in terms of wording, mathematical expressions, and alternative representations.

One major strategic choice for a textbook suggests the “macroscopic vs. microscopic” approach of the FLT (Tarsitani & Vicentini, 1996). While macroscopic approaches reflect the classical theory as developed during the 19th century (e.g. Baehr, 1973), the microscopic approaches introduce the statistical mechanical theory that was developed during the 20th century in a vastly mathematical way (e.g. Landau & Lifshitz, 1976). However, in some cases, elements of both approaches can co-exist in the same textbook (e.g. Young & Freedman, 2012). Another strategic choice for the architecture of a textbook refers to the “applications vs. axioms” approach of FLT (Christiansen & Rump, 2008). The first one unfolds the theory through a deductive methodology (e.g. Carathéodory, 1909), while the second gives prominence to the presentation of specific models and systems (e.g. Moran, Shapiro, Boettner, & Bailey, 2014).

The convergence of macroscopic-applications approaches is particularly evident in technical thermodynamics textbooks, that usually address novice and expert engineers. Such textbooks offer semi-quantitative energy distribution representations of theoretical and real engines, such as Sankey diagrams (e.g. Young & Freedman, 2012, p. 655). These representations are in-line with the historiographic analysis that illustrates the role of steam engines for the emergence of the FLT. At the same time, they allow the discussion to expand to crucial issues, such as engine efficiency and energy dissipation.

Cognitive analysis: students' alternative frameworks and reasoning patterns for the FLT

First- and second-year university students and upper secondary students hold similar conceptual frameworks regarding the FLT, as a result of their comparable reasoning patterns (Rozier & Viennot, 1991; Tiberghien, 1994). Students seek interpretations for the thermodynamic processes they encounter under the umbrella of linear causal reasoning. Linear causality leads them towards systematic neglect of variables, creation of preferential relations among variables and oversight of the symmetry between variables (Rozier & Viennot, 1991). Due to this reasoning pattern, students land on alternative frameworks that can belong in energy and/or non-energy frameworks.

The energy approach of a thermodynamic phenomenon through the FLT is adequate for the description of all thermodynamic processes. However, its use is limited and often inaccurate (Leinonen, Raesaenen, Asikainen, & Hirvonen, 2009; Meli et al., 2016). Within energy frameworks, students have difficulty in recognizing the energy entities that play a role in a thermodynamic process and also in identifying the increase or decrease of these entities (Meltzer, 2004; van Roon, van Sprang, & Verdonk, 1994). The above findings can be related to an inadequate representation of the thermodynamic system and its surroundings (Meltzer, 2004; van Roon et al., 1994).

In non-energy frameworks, students typically use the Ideal Gas Law or microscopic level approaches as stand-alone explanations or complementary to their perception of the FLT. The Ideal Gas Law is particularly favored, although it is inadequate in interpreting adiabatic processes. State entities are much more appealing to students than energy entities (Kautz, Heron, Loverude, & McDermott, 2005; Leinonen et al., 2009; Loverude, Kautz, & Heron, 2002; Meli et al., 2016) Micro-level descriptions are also used, as the “ultimate” explanation students can offer. However, due to their intrinsic complexity, it is difficult for students to handle them properly and provide an accurate and complete explanation for thermodynamic processes (Kautz, Heron, Shaffer, & McDermott, 2005; Leinonen et al., 2009; Meli et al., 2016; Meltzer, 2004). It should be noted that both the above non-energy frameworks are usually taught long before the FLT, undermining the exploratory power of the latter (Leinonen et al., 2009; Meli et al., 2016).

Pedagogical analysis: constructivism as a pedagogical approach of the FLT

Traditional instruction, in the sense of attempting to “transfer” knowledge to students, has poor learning outcomes in the field of thermodynamics (Meli et al., 2016). One feature of the traditional approach, that may influence these outcomes, is the presentation of many and diverse frameworks for the same section, without an in-depth analysis for any of them. Another feature is the “conveying” of this knowledge on an abstract basis, with no reference

to real-life situations, thus jeopardizing students' engagement to the subject (Koliopoulos & Constantinou, 2005).

To deal with these issues, constructivism proposes teaching that facilitates students in constructing their own meaningful, yet scientifically accurate, representations (Tiberghien, 1997) in a restricted framework with cultural context. For achieving this, three interrelated levels for the teaching and learning of science should be taken into account: phenomena, theory, and models (Tiberghien, Psillos, & Koumaras, 1995).

The phenomenological field of introductory thermodynamics mainly refers to thermodynamic processes. These should be organically integrated into a phenomenological field that is relevant to students, pertinent to what they usually face, free of technical challenges and capable of developing a “cognitive need” (Devi, Tiberghien, Baker, & Brna, 1996). The theory describing the phenomena should be both in a qualitative and a quantitative form. Qualitative form refers to the proper use of wording for describing entities and principles, while quantitative form refers to mathematical formulas and/or diagrams.

However, even if students form the mathematical expression of the FLT correctly, this does not necessarily reflect a sufficient understanding of the processes at hand (Kautz, Heron, Loverude, et al., 2005). The levels of phenomena and theory should be properly linked for students to seamlessly match phenomena to theory and the other way around. This link refers to the level of models that intervene between phenomena and theory to create smooth, yet meaningful, connections. For the FLT, the Energy Chain Model (ECM) can work towards this direction, since it suggests a semi-quantitative model representing energy distribution between the thermodynamic system and its surroundings during a process.

The convergence of the epistemological, cognitive and pedagogical analysis for the design principles of the TLS

We make decisions informed by the above-described three-dimensional analysis (Ruthven et al., 2009) and we seek intersections that should lead to a coherent design of the TLS and also a feasible implementation of this TLS for the upper secondary school level. Based on the threefold “theory-phenomena-models” as the levels for the teaching and learning of science, our main conclusions unfold respectively as follows:

- (a) *We exclusively employ the macroscopic (classical) approach of thermodynamics.* The epistemological analysis illustrates that this framework can efficiently describe the theory and applications of thermodynamics, as it was historically developed before the appearance of the microscopic (statistical) one and it often stands alone in contemporary textbooks (Meli & Koliopoulos, 2019). Cognitive analysis confirms the appropriateness of this choice for novice learners, as the microscopic approach is the source of various misinterpretations and it hinders the energy-related explanations of thermodynamic processes (Kautz, Heron, Loverude, et al., 2005; Leinonen et al., 2009). Finally, sticking to a single framework and deeply examining it is consistent with the pedagogical analysis (Koliopoulos, Adúriz-Bravo, & Ravanis, 2011).
- (b) *We organically integrate the steam engine as a study object.* Historiographical analysis suggests that steam engines had a great contribution to the development of thermodynamics (Cardwell, 1971; Kuhn, 1977), while textbook analysis shows that they used to be and still are a standard component for physics and engineering education. Employing them as our basic phenomenological reference, students'

frameworks can be more accurate and energy-oriented (Cochran & Heron, 2006). Additionally, steam engines bring thermodynamic processes in a real-life context, within which students can find common ground with the respective theory (Koliopoulos et al., 2011).

- (c) *We give prominence to energy distribution representations of thermodynamic processes, through the ECM.* Epistemological analysis confirms the representational power of such models, especially in reference to steam engines. From a cognitive point of view, these representations are in-line with students’ intuitive linear causal reasoning pattern (Rozier & Viennot, 1991), but they can also pave the road for students to overcome it. Versions of the ECM, like the one we propose, can work efficiently towards the meaningful connection between the levels of theory and phenomena (Delegkos & Koliopoulos, 2018; Tiberghien, 1996).

CONTENT OF THE TEACHING AND LEARNING SEQUENCE

Table of contents of the TLS

The TLS consists of three broad-sections (BS) that include twelve sub-sections (SS). Throughout the TLS we examine Newcomen’s steam engine and we focus on the gas in its motor as an example of a thermodynamic system. On this account, we introduce the FLT as an energy explanation for the description of adiabatic, isothermal and isobaric processes. Lastly, we use these components to assemble the cyclic process and discuss the efficiency of the engine. Several additional elements are also examined during this course, such as the properties of thermodynamic systems and the Carnot cycle.

Table 1 presents the contents of the TLS. The “activity problem” column describes the issue students attempt to address throughout each sub-section. The “conceptual components” column refers to the thermodynamic concepts and processes that are being introduced in a sub-section. Finally, the “conceptual negotiation” column mentions the representations that students are expected to construct during the instruction of the sub-section.

Table 1. Sample of the TLS content table.

	ACTIVITY PROBLEM	CONCEPTUAL COMPONENTS	CONCEPTUAL NEGOTIATION
BS1	Thermal engines and thermodynamic explanation		
SS01	What is a thermal engine and how does it work?	Structure/function of Newcomen engine	Activation of pre-energy conceptions
SS02	How can we explain the function of the Newcomen engine in terms of energy?	Work and heat	Qualitative representation of energy distribution Semi-quantitative energy distribution
SS03	How does the Newcomen motor work?	Thermodynamic system and surroundings	Introduction to the ECM
BS2	Gas processes and the first law of thermodynamics		
SS04	Why did an ignition take place in the tube?	Adiabatic compression/ Change in internal energy	Qualitative and quantitative representation of energy distribution and conservation: <ul style="list-style-type: none"> • phenomenon → ECM • phenomenon → ECM → mathematical expression • mathematical expression → ECM → phenomenon
SS05	Why was the air in the tube liquified?	Adiabatic expansion/ Work/ FLT	
SS06	Why did the piston move (I)?	Isothermal expansion/ Heat/ FLT	
SS07	Which alternative phenomenon does the expression on isothermal expansion describe?	Isothermal compression/ Heat and work/ FLT	

SS08	Why did the piston move (II)?	Isobaric cooling/ FLT	Distinction between heat and change in internal energy Overcoming linear causality
SS09	What will happen if we isobarically heat the gas?	Isobaric heating/ FLT	
BS3	Improving a thermal engine: a historical issue		
SS10	Which processes take place in Newcomen motor?	Cyclic processes	Qualitative and quantitative representation of energy distribution in complex systems Generalization of FLT as an expression of energy conservation
SS11	What is the efficiency of a Newcomen engine?	Real engine efficiency	
SS12	How can an engine's efficiency surpass Newcomen's?	Theoretical engine efficiency	Overcoming linear causality: energy dissipation

Presentation of a TLS sub-section

In Table 2, we take as an example SS3 to briefly describe the way the TLS is implemented. The first two columns (“teacher’s activities” and “students’ expected activities”) refer to practical issues, while the last two columns (“conceptual components” and “conceptual negotiation”) address the respective theoretical issues for this particular sub-section.

Table 2. Sub-section 3 “How does the Newcomen motor work?”.

TEACHER'S ACTIVITIES	STUDENTS' EXPECTED ACTIVITIES	CONCEPTUAL COMPONENTS	CONCEPTUAL NEGOTIATION
Presentation: Animation of the engine in operation		Structure and function of a steam engine	
Activity sheet: Which part is the motor of the engine?			
Discussion: Motor as a thermodynamic system/ system's surroundings	Discuss the concepts of the system and its thermal-mechanical surroundings	Thermodynamic system and thermal-mechanical surroundings	Construction of the concepts of system and surroundings / Semi-quantitative ECM
Activity sheet: The motor's cycle is one uniform process or several diverse processes?			
Discussion: Components of a cyclic process - examples	Discuss the features of a cyclic process	Cyclic process in a steam engine	Construction of a qualitative approach of the cyclic process

RESULTS FROM THE IMPLEMENTATION OF THE TLS

In this paragraph, we briefly describe the results of the research in reference to the conclusions that were drawn from the convergence of the epistemological, cognitive and pedagogical analysis. This TLS was implemented as a case study in a class of 19 students of the second grade of upper secondary school in Athens (Greece). The results reported here derive from the qualitative and quantitative analysis of pre- and post-tests.

- The restriction of the TLS in macroscopic thermodynamics and with special focus on the energy framework was very much reflected in students' final explanations of thermodynamic processes. While in the pre-test they used a variety of alternative frameworks, including the microscopic one in significant percentages, in the post-

test almost every student employed the FLT framework and many among them provided explanations at a high-efficiency level.

- (b) Newcomen's steam engine suggested a functional phenomenological field for the introduction and application of the FLT. The quest for the operation of the engine facilitated the organic integration of several thermodynamic processes; through this real-life situation, representations that otherwise would be abstract, became meaningful and approachable. In the post-test, students provide energy explanations for thermodynamic processes by assessing their utility in the engine's motor.
- (c) The ECM as a representation for the energy distribution between the targeted thermodynamics system (gas in the engine's motor) and its surroundings was at great extend used by students for explaining thermodynamic processes in the post-test. Therefore, it appears as a proper model for bridging the phenomena to the theory of the FLT that describes them.

DISCUSSION

For the design of a TLS addressing the FLT for the upper secondary school level, we thoroughly analyzed preceding works on the field of introductory thermodynamics (Ruthven et al., 2009). This analysis was unfolded in three different domains: epistemology, cognition, and pedagogy (Buty et al., 2004). We drew our basic design principles for the TLS through the convergence of these domains. All parts of the analysis agreed on the use of (a) the macroscopic thermodynamics approach (Meli & Koliopoulos, 2019), (b) steam engines (Cochran & Heron, 2006) and (c) energy distribution models (Koliopoulos et al., 2011). Based on these design principles, we formed and implemented a medium level TLS (12 subsections) with related conceptual components and issues for negotiation with students, for them to improve their respective representations. The quantitative and qualitative analysis of pre- and post-tests confirmed our choice of design principles, as students' representations were in-line with them and their representations achieved satisfactory levels of sufficiency.

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