Mathematical problem solving in early education. Possibilities and prospects.

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Abstract

May problem solving be the object of teaching in early education? May appropriate teaching interventions develop as to scaffold children's efforts to solve mathematical problems? These are the central questions of this paper. The sample consisted of 18 children of a Cyprus public pre school classroom and the problem they were asked to solve was to find all solutions of the pentomino. Graphically representing the solutions on squared paper sheet supported the children's efforts.

The findings show that children responded positively to the problem and were successful in finding all solutions for the specific problem. Graphically representing the solutions as well as the forms of teacher-children interaction played an important role for the positive outcome of the activity.

Keywords: problem solving, graphical representations, pentomino, early education.

Introduction

An overview of fairly recent research findings in the area of preschool math education, (Greens et al., 2004) notes that young children (as young as 3 years-old) acquire informal forms of mathematical knowledge that are more complex than what was earlier believed. Moreover young children are able to learn more complex and sophisticated mathematics than what is suggested by preschool math programs. Today we can claim that there is a solid base of research findings that enables us to talk about young children's developed abilities in math understanding as well as the manipulation of situations whose confrontation demands solutions of mathematical problems (Ginsburg & Golbeck, 2004; Greenes et al, 2004).

This paper falls within the sphere of mathematical problem solving. The children were asked to find all possible answers of the pentomino problem. The teaching methodological development that is presented here focuses on two themes: problem solving and the development of practices concerning schematizing data as to facilitate the solution of the problem.

Mathematical problem solving processes and graphical representations

Nowadays, the development and acquisition of the ability to solve mathematical problems is placed among the basic aims of math education. More particularly, solving mathematical problems is an activity that requires the child's engagement in a variety of cognitive actions integrating knowledge and skill. (Cai & Lester, 2005, p.221).

A problem situation can be defined as a new activity meaningful to the child sufficiently close to his/her current knowledge on the one hand, but also sufficiently different on the other. The conditions which a problem situation should fulfill include: the child has to beginning, knowledge being constructed, the problem situation involving a research process with repeated trials, conjectures and verifications, the problem situation being self-correcting and the problem leading to the construction of new knowledge (Arsac et al 1988, in Grugnetti et al, 2005; Grugnetti et al 2005). The basic stages of the problem solving process can be defined as: problem formulation, solution planning, solution design, solution testing and evaluation (Deek et al, 1999; Kappa, 2002)

Representation as a term is usually distinguished between internal and external (Goldin & Kaput, 1996). Internal representations are defined as the set of intellectual pictures ideas and expressions of a person that enable him/her to connect data and situations and to assess and distinguish between principal and secondary data (Goldin & Kaput,1996). External representations include symbols, diagrams, figures etc. They constitute forms of the child's statements that enable us to observe the degree of his/her understanding of a specific notion. On the other hand, external representations such as diagrams, tables, graphs as well as conventional symbols used in mathematics, can play the role of stimuli of data transfer.

The framework of this study looks at external teacher and children representations as members of a specific context: the classroom during the process of mathematical problem solving.

It has been found that the graphical representation of data can be accessible to children as young as 5 years old, under the condition that such practices are introduced to the children's educational activities in

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a functional manner (van Oers, 1994, 2001). Through representations children have the opportunity to organize their thoughts and knowledge.

In the early stages of education children rarely resort to the use of written symbols and pictures in order to record their thoughts initiatively, particularly their mathematical ideas. Children do not usually understand why they should use this singular set of mathematical symbols. Research studies show that pre school and early elementary school children, although familiar with arithmetic symbols, usually avoid using them when recording quantities (Sinclair et al, 1983).

Nowadays mathematical education as early as preschool education encourages children to record data with notes, symbols and schemata. Children are often invited to record data dealing with quantity or space and solve mathematical problems using schematizing. Moreover, children are expected to interpret symbols and have the ability to use them during mathematical practices. This means that children should have the ability to reconstruct, interpret and transform related data in forms found within the science of mathematics.

As a conclusion, and recognizing the usefulness and importance of creating and using symbolic representations during the mathematical problem solving process, the teacher is called to answer to questions relative to the way she can support and encourage children to use symbolic representations.

Our aim is to explore the possibility of preschool children solving math problems. The specific problem that the children are asked to solve is to find all possible solutions of constructing a pentomino. Children were encouraged to use graphical representations of the solutions they find. The question we set out to answer is:

Can preschool children engage in a problem solving process, including graphical representation, systematically?

The study question shall be explored through the construction of the pentomino problem solutions.

Method

The problem

Children were asked to find all possible solutions to the pentomino problem. Each pentomino solution consists of a polynomial of 5 identical squares joined on one side (Fig.1). There are 12 different pentomino shapes which are usually named after letters of the Latin Alphabet to which they are similar. A shape created by reflection or rotation of the initial 12 solutions is not considered as a different solution.

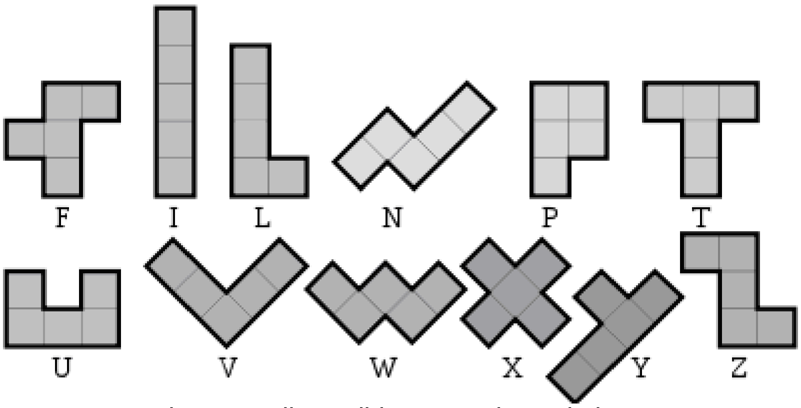

Figure 1. All possible pentomino solutions.

Participants

18 children of a Cyprus preschool public class aged 4-5.5 (av=4.8) took part in the study. The classroom teacher had the role of the researcher too.

The process

Within the interests of this study were the attitudes, skills and conceptual understandings developed by the children during the process of solving the pentomino problem, as well as the role of the teacher within this process.

An observational grid adapted to the aims of the current study was used. (Cohen, et al., 2004; Cai & Lester., 2005; Leikin & Kawass, 2005). The themes of observation taken under consideration were:

(I) observation of children's strategies while solving the problem

(II) the use or not of graphical representations during the problem solving process

(III) (in the cease of the use of graphical representation) the facilitation of graphical representations in the problem solving process

The data consisted of audio and video taped recordings of classroom episodes, teacher recordings, children's work sheets and the teacher's reflective diary.

Children worked individually while seated in a group of 4. The teacher encouraged them to interact with the other children sitting with them and they moreover, were free to do so if and when they wanted.

It is important to note that the children were not obliged to find all pentomino solutions in one session. They could stop and return the following day if they felt tired or wanted to engage themselves in a different activity. Each child could work on the pentomino problem for 3-4 consecutive days. The pentomino problem remained the main math problem of the math table for 4 months (February to June) and children could solve it over and over again if they chose to do so.

Materials

Children created the pentomino solutions using 5 identical plastic squares. They also had a sheet of paper with squares on which to record their answers, a basic element of the teaching intervention was to encourage children to represent their solutions on the squared answer sheet.

Data Analysis

The data was analysed and categorized as follows:

(I) Skills involved in solving the pentomino problem: While working on the pentomino problem, children had the opportunity to demonstrate a variety of mathematical conceptual understandings such as counting, addition and subtraction, graphic skills (showing their ability to manipulate constructions concerning space) etc.

(II) Interaction contributing to different forms of dialogue: During the initial phase of solving the pentomino problem children seeked their teacher's help, chose to work individually- with limited interaction with the children sitting at the Math table at the same time. But during the course of the process they altered their attitude. Some children (especially the older children) formed pairs. Some pairs created a kind of internal "differentiation" in their work: one child produced the solution using the squares and the other child recorded it on the answer sheet, while at the same time each observed the actions of the other intervening at each other's construction when they felt it was needed. It is interesting to note that they turned to their teacher mainly to announce an idea, or show a solution. Mostly, when faced with a difficulty, children would turn to another child rather than their teacher.

Interactions between children and between teacher and children were particularly effective in the problem solving process

(III) Graphical representations in the mathematical problem solving process: for recording the different shapes of the problem children used a squared answer sheet. Initially they found it difficult to record their constructions on paper due to their insufficient perception of space- more specifically in mapping each square of their construction to its corresponding square on the answer sheet. Frequently children would ask the teacher to mark the squares on the answer sheet for them. In these cases the teacher would express her way of thinking acting out loud

Given that the pentomino was the main mathematical problem occupying the Math table for a period of 4 months, children gradually, initially older children (age 5) and later younger children (age 4), started recording simpler pentomino constructions (e.g constructions I, L, T) and their asking for the teacher's help was limited to more complicated constructions. After the first successful recordings children felt more confident and tried persistently to create their own representations in later constructions.

Usually most children would imitate the recording process demonstrated by the teacher until they gained the autonomy to move to form their graphical representations alone or with the help of their peers. Of course there were children who developed their own, particularly effective, representation strategies. As demonstrated in Figure 2 the child uses her own strategy to represent solutions L and T using a holistic approach without marking the squares she is going to color, while for representing all other solutions she chooses to use the strategy demonstrated by the teacher: marking each square according to the solution found using the construction material.

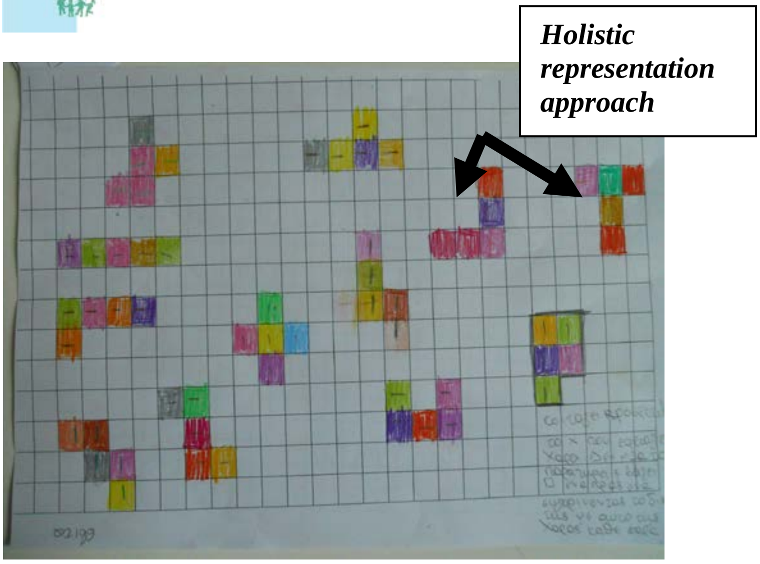

Figure 2: Graphical representation strategies

There were times when the children made mistakes in the number of squares they colored. The most frequent mistakes were recorded with construction "I" where instead of 5 squares they would color 6 and construction "T" where they would color 3 squares horizontally and 3 or 4 squares vertically (Figure 3).

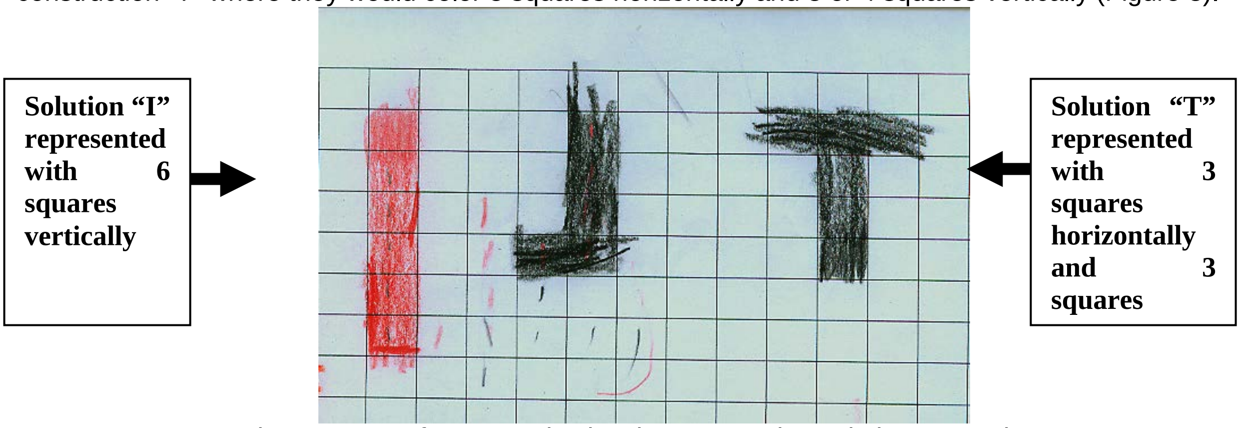

Figure 3: Most frequent mistakes in representing solutions "I" and "T"

During the final stages of the recording process when children gave constructions they'd already recorded with a different orientation. In these situations the teacher would ask the children to carefully look and study their records resulting in the children being able to locate the specific representation (Fig. 4)

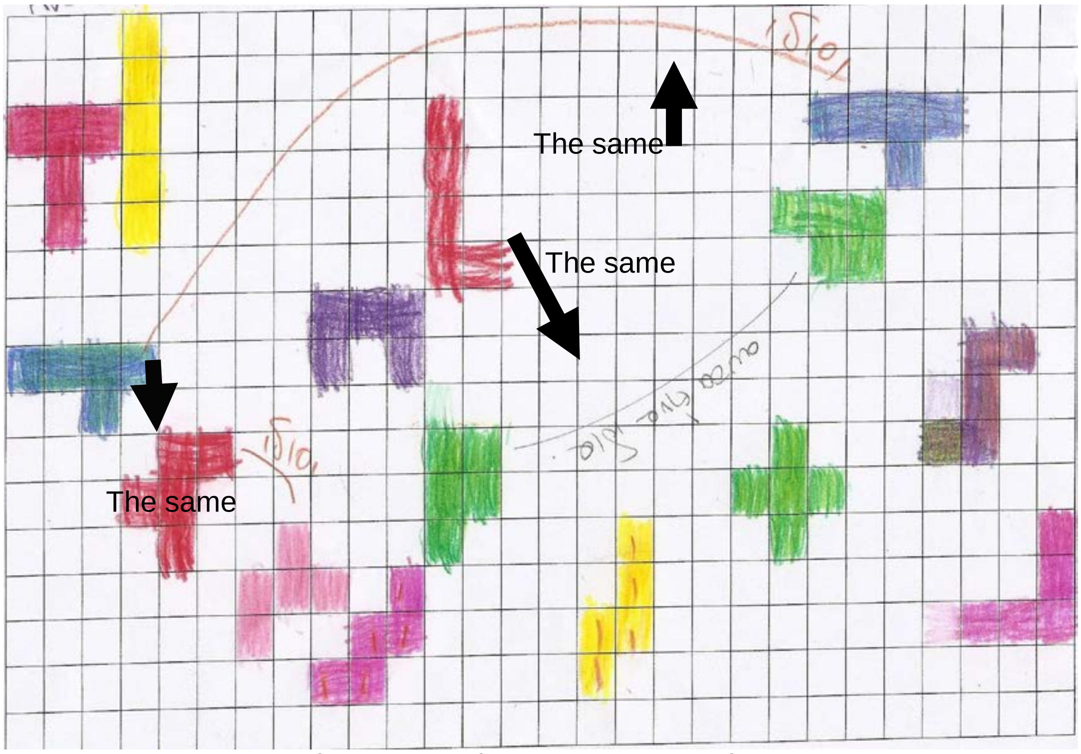

Figure 4: Locating same representations.

Gradually, children realized the usefulness of representations as an effective way of finding all possible solutions to the pentomino problem, since it was practically impossible to keep all solutions created with the material. Moreover, while initially they recorded each solution of the pentomino rather mechanically, after time and when they became more familiar with the process, they would carefully study their answer sheet before recording a solution to make sure the current one was original.
According to the principles of the pentomino problem, symmetrical constructions as well as identical constructions are not considered as solutions. Children had difficulty in identifying the similarity in a solution when it was rotated or flipped. For their familiarization with such solution similarities the teacher would draw a bold outline to that solution and turn the answer sheet 90, 180 or 270 degrees so that the children could see the different positions of the same solution and compare them to the one they had found (Figure 5).

Figure 5: Helping the children see the different positions of the same solution

Conclusion and discussion

In the current study we looked at preschool children's ability to respond to mathematical problem solving processes such as the pentomino problem. In their quest to find all 12 solutions to the problem, graphical representation of solutions on a squared answer sheet was proposed.

Concerning the study question, whether preschool children can successfully solve the pentomino problem and thus can mathematical problem solving be the object of teaching in preschool education- the majority of children responded positively. Of course this was the result of different factors. Firstly, the gradual development of the problem during a long period of time which enabled children to systematically work on the problem. Secondly, the teacher's contribution a) in enabling and supporting the development of children's autonomy and b) in organizing the classroom in a way which gave the possibility for creative interaction to be developed.

During the problem solving process children demonstrated numerous abilities such as the ability to count and add, the ability to identify same shapes differently positioned as well as their general ability to focus on a specific notion and cooperate with their peers in order to solve a problem.

Frequently children used mainly both forms of communication: verbal (during the initial stages) and non verbal communication (during later stages). Through verbal communication children helped each other by expressing their thoughts out loud. Through non verbal communication one child would turn to another child's answer sheet for new ideas. Moreover, when two or more children had difficulties in finding a solution the need of cooperation amongst them was created. In this case communication enclosed both above mentioned forms of communication. Cooperating children talked to each other and expressed their thoughts out loud while at the same time they used their recordings to reach the desired result. Cooperation for a common solution was found mainly in pairs of children, and almost always when children had a closer relationship beyond the boundaries of this specific problem. Children chose to work with children they felt more comfortable with.

A concise presentation of children's skills, the forms of interaction and the teacher's contribution is shown in Table 1

Demonstration of children's abilities	**Forms of interaction**	**Teacher practice**
• counting • addition, subtraction • same shape identification • graphical representation abilities • graphical abilities • focusing on a specific aim	• personal activity • team activity, mainly in pairs • teacher-child interaction	• guiding children • supporting children autonomy • organizing alternative ways of action (personal work, group work, classroom work)

Table 1: Skills, forms of interaction and pedagogical practices

As earlier stated in this paper occupy in problem solving organization when the process needed to be followed is new and at the same time the problem itself has a complexity arousing their interest. Research findings (Kapa, 1999, in Kappa 2002) in this area show that children organize the way they solve problems using 3 basic strategies: a) trial and error, b) step-by-step planning and c) holistic planning. In our study it is clear that most children initially tackled the pentomino problem using the trail and error method. When they became more familiar with the problem, children followed step-by-step planning as well as more refined strategies.

Concerning the contribution of graphical representations in the process of problem solving findings suggest that initially children faced difficulties in graphically representing the solutions of the problem. They were able to overcome these difficulties gradually through the teacher's interventions.

This fact underlines the importance needed to be given by math education to children's familiarization with graphical representations as well as the development of their ability to assess graphical representations (why is this graph important? Why is a graph useful? Which type of graph is suitable in a certain situation for solving a certain problem?). Our findings also lead to the conclusion that the initial mechanistic graphical representations of solutions was gradually replaced by a conscious effort to find and record original solutions (Figure 3).

The role of the classroom context is substantial in the process of problem solving in preschool (Zacharos et al., 2011). Similarly, graphical representations activities in preschool education need to be embedded

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within a context which enables us to expect children's positive response. Such a context is functional for children's learning when it encourages children's interactions.
The teacher's role in the above described learning process proves to be detrimental in guiding children, scaffolding their autonomy and providing multiple interaction experiences. Classroom organization, indirectly reflecting the role of the teacher, gave the children the opportunity to pass from individual work to group work.

Acknowledgement

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